

All-optical noise quenching of an integrated frequency comb

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Integrated frequency combs promise transformation of lab-based metrology into disruptive real-world applications, particularly with octave-spanning microcombs enabling self-referenced optical synthesis and clock implementations. However, the integrated resonators that support microcombs suffer from thermal fluctuations, limiting microcomb use outside laboratories due to the need for bulky feedback systems. Kerr-induced synchronization (KIS) offers a solution by eliminating this electronic servo control through all-optical locking. Here, we show how KIS profoundly alters the noise characteristics of soliton microcombs and enables a small device footprint to be compatible with low-noise operation. The phase locking between the dissipative Kerr soliton (DKS) and the injected reference reduces the tooth-to-tooth pump noise propagation—enabling easier carrier-envelope offset stabilization and uniform spectral performance—while also damping the impact of intracavity fluctuations on the DKS, such as thermo-refractive noise (TRN). Our theoretical and experimental results show that KIS modifies the comb noise elastic-tape model, maintaining comb tooth linewidths comparable to the pump lasers', unlike single-pumped systems where linewidths increase by orders of magnitude from the comb center to its edges. Additionally, KIS quenches intrinsic noise sources at the soliton decay rate, regardless of laser coherence or microring thermo-refractive correlations. Using these findings, we demonstrate an octave-spanning microcomb operating below TRN limits, using both free-running lasers and lasers correlated via comb self-referencing, with performance limited only by laser frequency noise. © 2025 Optica Publishing Group under the terms of the Optica Open Access Publishing Agreement

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1. INTRODUCTION

Optical frequency combs are essential components in the metrological toolbox because they provide a coherent optical-to-microwave frequency link [1], enabling accurate optical frequency measurement [2–5], which has found application in optical frequency synthesis [6], spectroscopy [7], microwave signal generation [8], and ranging [9], among other critical applications [10–12]. To further lower the size, weight, and power consumption (SWaP) of combs for deployable applications, on-chip microcombs can be created by taking advantage of nonlinear photonic devices. By leveraging the third-order nonlinearity ($\chi^{(3)}$), integrated microring resonators can be designed to support dissipative Kerr soliton (DKS) states that exist by doubly balancing the loss/gain and dispersion/nonlinear phase shift of the system [13,14], which once periodically extracted create a uniformly spaced pulsed train, and hence an on-chip frequency comb. While many applications of table-top frequency combs have been reproduced at the chip scale thanks to DKSs [15–20],

these integrated microcombs fall short in their repetition rate noise performance—primarily due to their poor ability to manage the cavity thermo-refractive noise (TRN) [21–24], which increases inversely with the resonator size [25]. Since TRN locally modifies the material refractive index, the cavity soliton experiences transduction of this noise onto both its group and phase velocity, which respectively impact the repetition rate and the carrier envelope offset (CEO) of the microcomb [Fig. 1(a)]. This noise transduction is particularly consequential for octave-spanning microcombs, where CEO detection and stabilization is a focus, and where SWaP considerations favor a high repetition rate to maximize individual comb tooth power within an octave, resulting in the use of a small microring resonator [20,26–28]. Mitigation of TRN is possible, for instance, through judicious use of a low power cooler-laser that can squeeze the resonator temperature fluctuations [29], leading to an improvement in the repetition rate noise by about an order of magnitude [21]. Other promising methods, such as dispersion engineering to tailor the recoil and counterbalance TRN, have been proposed [30] and could potentially fully quench

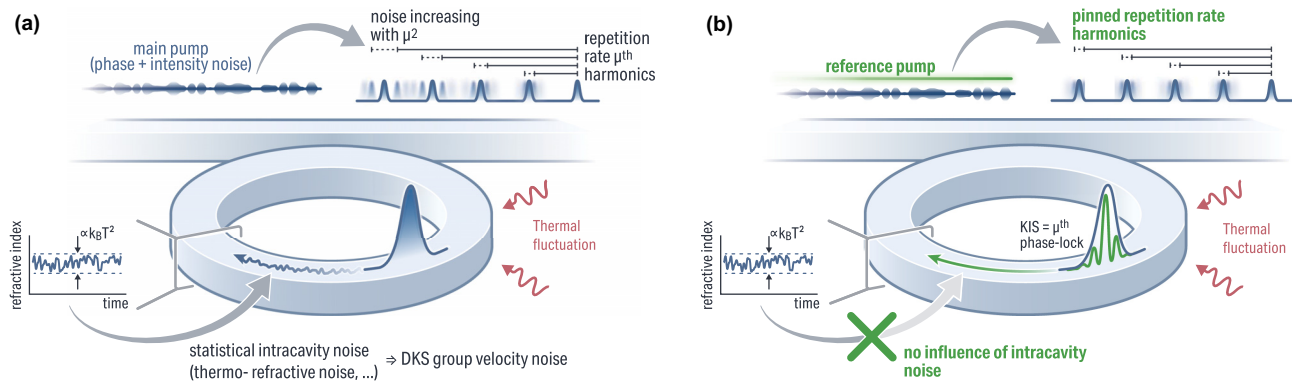


Fig. 1. DKS thermo-refractive noise and its circumvention through Kerr-induced synchronization (KIS). (a) Single-pump scenario. The statistical fluctuation of the cavity temperature modifies the cavity material refractive index, which in turn impacts the group velocity of the cavity soliton and hence the repetition rate of the generated frequency comb. In addition, the frequency noise of the pump is transduced onto the cavity soliton group velocity, resulting in noise that increases quadratically between one pulse and another separated by harmonics of the repetition period. In the frequency domain, this corresponds to increasing noise as a function of comb tooth mode number μ with respect to the pump, which propagates down to the carrier envelope offset (CEO). (b) In Kerr-induced synchronization, a reference is sent to the microcavity where the DKS lives, resulting in a phase locking of the cavity soliton to the reference phase velocity. Since the soliton repetition rate is now determined by external parameters that are the two pump frequencies, intrinsic noise sources, such as TRN, are bypassed and no longer affect the DKS repetition rate. In addition, the microcomb is now dual-pinned at the comb tooth closest to the reference and at the main pump, resulting in noise propagation onto the comb teeth that is damped relative to the single-pump case, directly impacting the linewidth of the CEO.

it. Yet, such engineering is challenging for octave-spanning combs since the dispersion would need to simultaneously support the desired microcomb bandwidth and the required asymmetry for TRN suppression. A final solution is to strongly suppress TRN by working at cryogenic temperature, which reduces the thermo-refractive coefficient by more than two orders of magnitude [31], and has been demonstrated to enable direct and adiabatic DKS generation [32]. Though very effective at removing TRN and other vibrational-based noise sources (e.g., Raman scattering), the significant infrastructure requirements are incompatible with fieldable applications. Beyond repetition rate noise, the individual comb tooth linewidths, as well as the carrier envelope offset tone linewidth, which are directly related to the noise of the main pump laser generating the DKS through a so-called elastic-tape model [33], can also be a limitation. Here, the frequency noise power spectral density (PSD) of the pump laser propagates quadratically with the comb tooth mode number—which in the temporal domain corresponds to a quadratic increase of noise between pulses separated by harmonics of the repetition period [Fig. 1(a)]—and broadens the comb teeth beyond the linewidth of the pump laser. For octave-spanning combs, where comb teeth that are hundreds of modes away from the pump are used (e.g., in self-referencing), this can be a significant limitation.

Recently, a new method to stabilize the DKS has been proposed, called Kerr-induced synchronization (KIS) [28]. KIS relies on the phase locking of the intracavity soliton to an external reference laser injected into the same resonator where the soliton circulates, which locks their respective CEOs [Fig. 1(b)]. This method has been demonstrated to be highly versatile, as any comb line can be synchronized [34], and multicolor synchronization that stabilizes the common repetition rate can also be achieved [35]. It allows for a passive and all-optical stabilization of the microcomb to an optical frequency reference, providing long-term stability for clockwork applications [28] with optical frequency division of the pumps' noise onto the repetition rate [34–36]. However, the limit of KIS on the noise performance of the microcomb has

yet to be determined, and only the impact of the amplitude of either the reference or comb tooth power on the KIS dynamics has been explored. Mathematically, the injection of another laser into the DKS's resonator allows for the existence of a new attractor, regardless of its coherence or TRN anti-correlation with the main pump driving the DKS, which is reached when entering the KIS regime. Since attractors in nonlinear systems only occur when they are dissipative, not only does the amplitude of the energy exchange matter—like in previous studies—but so does the characteristic time scale of the energy exchange, which, in our case, is the photon lifetime.

Here, we theoretically analyze and experimentally demonstrate the profound improvements that KIS enables on both the repetition rate noise and individual comb tooth noise in DKS microcombs. We show that the addition of a second intracavity field through the reference critically modifies the elastic-tape model describing the increase in the comb tooth linewidth away from the main pump such that under KIS, all comb tooth linewidths remain within the same order of magnitude across a span of more than 200 comb lines. Consequently, the free-running CEO tone linewidth in KIS decreases by two orders of magnitude compared to the single-pump microcomb. In addition, we demonstrate theoretically and experimentally that synchronization through the reference laser changes the soliton's dynamics such that any internal cavity noise, and in particular TRN, is damped at a rate proportional to the photon lifetime. This behavior, which is expected for dissipative system attractors, essentially bypasses current limitations on microcomb noise performance [Fig. 1(b)] and enables any cavity size to perform at the same noise level as a large resonator. We experimentally verify our numerical and theoretical analyses using an octave-spanning integrated microcomb with a volume of $\approx 80 \mu\text{m}^3$, for which we obtain lower repetition rate noise than the TRN-limited value while using free-running and CEO-locked pumps.

2. INDIVIDUAL COMB TOOTH LINEWIDTH REDUCTION IN KIS

First, we will discuss the optical linewidth of the individual teeth forming the comb, which is critical in applications such as optical frequency synthesis and spectroscopy, where having narrow individual comb teeth is essential. As discussed previously, we will focus on free-running operation, where two main noise sources from the DKS pump laser can be considered: frequency noise and residual intensity noise (RIN); laser shot noise will be neglected since the frequency noise will be orders of magnitude larger when not locked to a stable reference cavity. Other intrinsic noise of the microring resonator, such as TRN, will be discussed later in this paper.

In the single-pumped DKS, noise propagation from the pump, either from its frequency noise or RIN, to the microcomb teeth follows the well-known elastic-tape model [33]. Interestingly, the frequency noise must account for soliton recoil [23], which can appear from the Raman self-frequency shift [37,38], and/or dispersive wave-induced rebalancing where the center of mass of the DKS must be null [39]. Here, the Raman effect is neglected since its impact is marginal in Si_3N_4 compared to other materials [40], and we assume that the recoil mostly comes from the imbalance of the DW powers creating a shift of center of mass. The PSD of a comb tooth μ , referenced to the main pump ($\mu = 0$), can be written as $S^{1p}(\mu, f) = S_{\text{orin}}^{1p}(\mu, f) + S_{\omega}^{1p}(\mu, f)$, with

$S_{\text{orin}}^{1p}(\mu, f) = S_{\text{orin},p}(f)\mu^2$, where $S_{\text{orin},p}$ already accounts for the power-to-frequency noise transduction from the pump and the frequency noise PSD defined as described in Ref. [23] as

$$S_{\omega}^{1p}(\mu, f) = S_{\omega,p}(f) \left(1 - \frac{\partial \omega_{\text{rep}}}{\partial \omega_p} \mu \right)^2, \quad (1)$$

where $S_{\omega,p}$ is the frequency noise PSD of the main pump, ω_{rep} is the repetition rate, and ω_p is the main pump frequency. Therefore, the main pump noise cascades onto the comb teeth at a quadratic rate [Fig. 2(a)]. The superscript $1p$ refers to the single-pump DKS operation, which will be compared against the KIS regime. It thus becomes obvious that, apart from a quiet comb tooth at $\mu_q = (\partial \omega_{\text{rep}} / \partial \omega_p)^{-1}$, the noise propagation makes the individual comb teeth broader than the pump itself, in particular for teeth far from the pump. Such comb teeth are of critical importance in octave-spanning combs; this noise propagation is a hindrance to performance.

Since KIS involves phase locking of the frequency components at the reference mode μ_s , resulting in the capture of the comb tooth closest to the reference pump laser, the noise propagation from the pump lasers will be largely different from the single-pump regime [Fig. 2(b)]. First, the microcomb is now fully defined by the frequency of both lasers, regardless of their power. Thus, as long

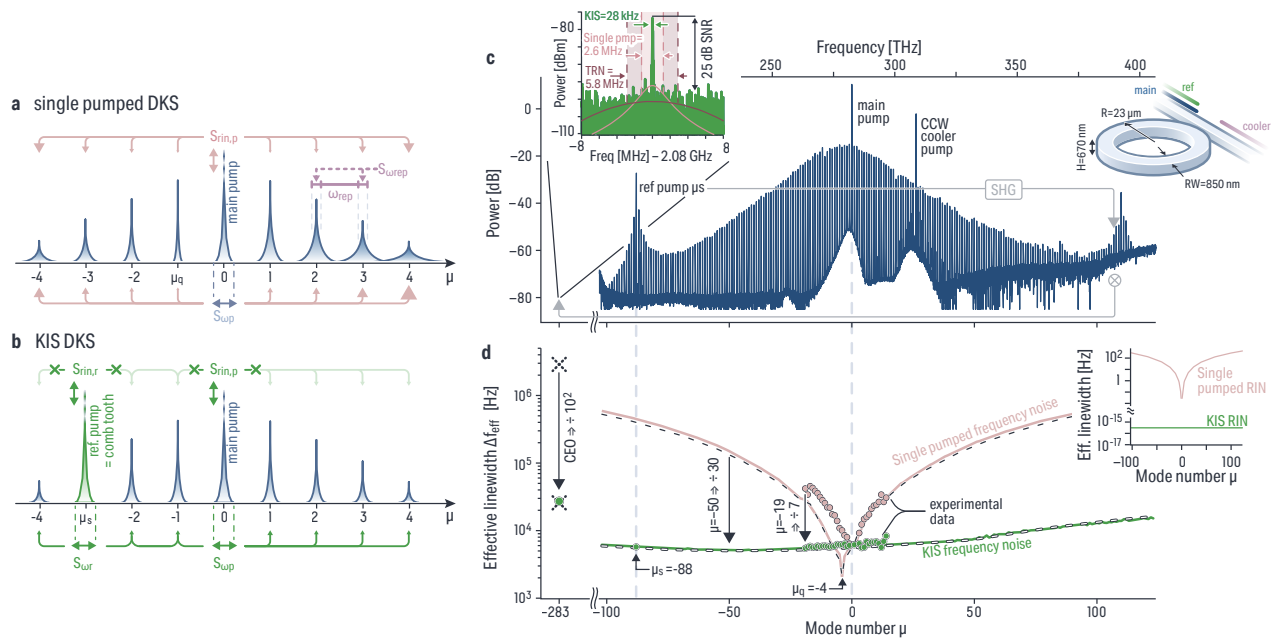


Fig. 2. Impact of KIS on the individual comb teeth linewidths. (a) Schematic of noise propagation of the single-pumped microcomb following the elastic-tape model. (b) In KIS, dual-pinning of the comb prevents any RIN transduction onto the comb teeth frequency noise while also greatly limiting the propagation of each pump's noise, $S_{\omega,p}$ and $S_{\omega,r}$, onto the comb teeth. (c) Optical spectrum of the experimental microcomb obtained with a main pump at $\omega_{\text{pmp}}/2\pi \approx 282.4$ THz (1061 nm) and the reference at $\omega_{\text{ref}}/2\pi \approx 194.5$ THz (1540.1 nm), from a Si_3N_4 microring resonator with dimensions $R = 23 \mu\text{m}$, $\text{RW} = 850$ nm, and $H = 670$ nm (inset). We use a cooler pump at $\omega_{\text{cool}}/2\pi \approx 308.7$ THz (971.8 nm) for DKS access. By doubling the reference (which is now a comb tooth) and beating it against a comb tooth, we retrieve the carrier envelope offset (inset) with 25 dB SNR (recording bandwidth of 20 MHz). The experimental CEO in KIS (green trace in upper left inset) agrees with the theoretical model. Assuming the same PSD, KIS enables far larger SNR than without KIS (pink curve in inset) or accounting for TRN (dark red curve) as in Ref. [21], allowing for the CEO to be locked. (d) Effective linewidths of the individual comb teeth from analytical prediction (dashed lines), LLE simulation (solid lines), and experimental data (circles) with free-running laser pump(s). In the single-pump case, the pump frequency noise propagates quadratically with the comb tooth mode number μ , exhibiting a quiet point at $\mu = -4$. In contrast, in KIS, noise propagation is damped, as the comb tooth frequency noise must match that of each pump at $\mu = 0$ and $\mu_s = -88$, due to the dual-pinning of the comb by the two lasers. The experimental data match well with the KIS model, while a discrepancy is observed for the single pump that is mostly related to thermo-refractive noise that is not yet included in the model. The elastic-tape model can be expanded to CEO ($\mu = -283$), leading to two orders of magnitude linewidth reduction between KIS and the expected single-pump case. Right inset: LLE simulation with the pump(s) relative intensity noise (RIN), whose effect on the linewidths is suppressed in KIS.

as the cavity soliton remains within the KIS bandwidth, which is dependent on the DKS μ_s modal component and reference intracavity energies [28,41] and is ≈ 1 GHz here, the impact of RIN from both pumps on the individual comb teeth will be entirely suppressed, such that $S_{\omega, \text{rin}}^{\text{kis}}(\mu) = 0$ [Fig. 2(d) inset], where the kis superscript refers to the KIS regime throughout the manuscript. Hence, in our approximation to dismiss for the moment the micro-ring intrinsic noise, the only contribution to the individual comb tooth noise PSD $S^{\text{kis}}(\mu)$ comes from the frequency noise cascading of both pumps. Since the captured comb tooth and the reference pump became indistinguishable in KIS, the comb tooth frequency noise is now pinned at each of the two pumps. In addition, the repetition rate noise must be the same for any two adjacent comb teeth considered. Hence, the noise propagation must also follow an elastic-tape model, this time adjusted for dual-pinning such that

$$S_{\omega}^{\text{kis}}(\mu, f) = S_{\omega, p}(f) \left(1 - \frac{\mu}{\mu_s}\right)^2 + S_{\omega, r}(f) \left(-\frac{\mu}{\mu_s}\right)^2, \quad (2)$$

with $S_{\omega, r}$ the frequency noise PSD of the reference pump and μ_s the comb tooth order undergoing KIS. It is interesting to compare the frequency noise in both regimes. Assuming the same frequency noise for both pump lasers $S_{\omega, p} = S_{\omega, r}$, and that $S_{\omega, \text{rin}}(\mu) \ll S_{\omega}(\mu)$, which is obviously true in the KIS case and has been demonstrated in the single-pump regime [23], one will observe $S_{\omega}^{1p} < S_{\omega}^{\text{kis}}$ for the modes $\mu = [2\mu_s \mu_q (\mu_s - \mu_q) / \mu_s^2 - 2\mu_q^2; 0]$. One of the main advantages of KIS is the possibility to dual-pin the microcomb's widely separated modes [28,34], while the recoil responsible for μ_q is usually close to the main pump. Hence, assuming $\mu_s \gg \mu_q$, the single-pump comb teeth PSD will only be better than the KIS one for a range $\mu = [2\mu_q; 0]$, which by definition is narrow since μ_q is small. By contrast, the KIS microcomb will exhibit better single-comb line frequency noise PSD performance across a broadband spectrum. Interestingly, in KIS when $S_{\omega, p} \approx S_{\omega, r}$, a new quiet point at $\mu_q = \mu_s S_{\omega, r} / (S_{\omega, p} + S_{\omega, r})$ will be present with a PSD reduction of twice the pump's PSD average. However, in the case that $S_{\omega, r} \gg S_{\omega, p}$ ($S_{\omega, r} \ll S_{\omega, p}$), the lowest comb tooth noise will be at the main pump $\mu_q = 0$ (reference pump $\mu_q = \mu_s$).

To demonstrate such performance improvement through KIS, we use an integrated microring resonator made of $H = 670$ nm thick Si_3N_4 embedded in SiO_2 , with a ring radius of $R = 23$ μm and a ring width of $RW = 850$ nm. This design provides anomalous dispersion around 283 THz (1060 nm) with higher-order dispersion providing nearly harmonic dispersive waves (DWs) at 194 and 388 THz, corresponding to $\mu = -88$ and $\mu = 110$, respectively [Fig. 2(c)]. We measure the individual comb lines from $\mu = -19$ to $\mu = 14$ using a high-rejection and narrow-band optical filter and an optical frequency discriminator (see Supplement 1 for details), while both main and reference pumps remain free running—possible thanks to the large KIS frequency window making the synchronization operation tolerant to short- and long-term frequency drift—with an effective linewidth $\Delta f_{\text{eff}} \approx 5.8$ kHz determined from the laser PSD measurement following the definition [42] $\int_{\Delta f_{\text{eff}}}^{\infty} S_{\omega}(f) / f^2 df = 1/\pi$. In the KIS regime, the measured comb teeth effective linewidths remain in the 5–7 kHz range, obtained from their frequency noise spectra in a similar fashion as the pumps. The experimental data closely match Lugiato–Lefever equation (LLE) simulations accounting for both pumps [43], which in addition match the dual-pinned elastic-tape model introduced above. Here, the experimentally

measured frequency noise PSD for each pump is input into the LLE simulation. Such close agreement between the model and the experimental data enables us to predict a comb tooth linewidth that will not exceed $\Delta f_{\text{eff}} = 15$ kHz for modes as far as $\mu = +120$. In sharp opposition, the predicted noise of the single-pump case, which as expected from previous work [23,33] matches between the LLE and analytical model assuming a quiet comb line at $\mu_p = -4$ obtained from the maximum of the sech^2 envelope of the experimental microcomb, yields an effective linewidth above 50 kHz ($10 \times$ that of the main pump) for modes below $\mu = -28$ or above $\mu = 22$, and as high as 500 kHz for modes close to the DWs. Experimentally, we confirmed such a trend, where Δf_{eff} increases with separation from the main pump, with $\Delta f_{\text{eff}} \approx 40$ kHz for $\mu = -19$. However, we do not observe a quiet mode in the measurements, and instead, all comb lines exhibit higher noise than the pump. This discrepancy, which has been observed previously [23], is mostly related to TRN that is not yet accounted for in the model. The lack of impact of its absence in modeling the KIS results already hints at its quenching in that regime, while it is the predominant noise source in the single-pump case.

Importantly, the modification of the elastic-tape model enables a quantitative comparison of the carrier envelope offset (CEO) linewidth. Under KIS, the CEO linewidth is reduced by approximately two orders of magnitude compared to the single-pump case, from 2.6 MHz to 27.1 kHz, assuming only the frequency noise of the pump. We experimentally confirm this through $f - 2f$ interferometry between the doubled reference laser (now a comb tooth) and its nearest comb tooth, measuring a KIS CEO linewidth of 28 kHz [Fig. 2(c) inset]. This drastic reduction in the CEO linewidth enables a larger signal-to-noise ratio and easier CEO locking, as we will discuss later. Additionally, we can estimate the CEO linewidth induced by thermo-refractive noise (TRN) to be approximately 5.8 MHz, using the system parameters presented in Table S.II of Supplement 1 and following the analysis in [21]. This also suggests that TRN plays a significant role in the single-pump case, which is largely mitigated in the KIS case. Additionally, LLE simulations confirm our intuition that pump RIN has been completely removed from influencing the effective linewidths of the comb teeth in the KIS regime [Fig. 2(d) inset]. In contrast, pump RIN is a non-negligible contribution in the single-pump case, although it remains orders of magnitude lower than the pump laser frequency noise. Such modification of the elastic-tape model for comb tooth noise could also provide valuable insights into other systems where similar injection locking of the comb tooth occurs. Recently, it has been demonstrated that recycling a comb tooth for self-injection locking back into the comb [44] results in a reduction in the comb teeth linewidths over a large modal span, akin to what is presented here. A similar analysis and modification of the elastic-tape model, like the one we proposed, could be used to predict the comb teeth linewidths in such a system.

3. THEORETICAL STUDY OF INTRINSIC NOISE SUPPRESSION IN KERR-INDUCED SYNCHRONIZATION

Since we observed good agreement between KIS experimental data and the theoretical dual-pinned elastic-tape model that only accounts for pump frequency noise, even with mode separation down to CEO, while the single-pump case presents a discrepancy that can likely be attributed to TRN, we now focus on studying

the microring resonator's intrinsic noise and how KIS modifies its impact on the cavity soliton. With this in mind, we study noise of the repetition rate ω_{rep} of the comb rather than noise of individual comb lines, since we have shown that regardless of operating in KIS or the single-pump regime, noise propagates through an elastic-tape model. We proceed with a linear stability analysis of the LLE for the cavity soliton outside of synchronization (i.e., akin to single-pump operation) and in the KIS regime.

Our starting point is the multidriven LLE (MLLE) [43,45], normalized to losses (see Supplement 1 for details), which can be written as

$$\begin{aligned} \frac{\partial \psi}{\partial t} = & -(1 + i\alpha_{\text{pmp}})\psi + i \sum_{\mu} \mathcal{D}(\mu) \tilde{\Psi} e^{i\mu\theta} + i|\psi|^2\psi \\ & + F_{\text{pmp}} + F_{\text{ref}} \exp[i(\alpha_{\text{ref}} - \alpha_{\text{pmp}} + \mathcal{D}(\mu_s))t - i\mu_s\theta], \end{aligned} \quad (3)$$

where ψ is the normalized intracavity field of the DKS following Ref. [43] and detailed in Supplement 1, θ is the azimuthal coordinate of the ring, μ is the azimuthal mode number, $\tilde{\Psi}$ is the Fourier transform of ψ from θ to the μ domain, t is the normalized time, $\alpha_{\text{pmp}}(\alpha_{\text{ref}})$ is the normalized detuning of the primary (reference) pump, $F_{\text{pmp}}(F_{\text{ref}})$ is the intracavity power of the primary (reference) pump, $\mathcal{D}(\mu)$ is the normalized integrated dispersion of the cavity as a function of relative mode number μ , and μ_s is the mode number of the reference pump in KIS.

In general, Eq. (3) admits multicolor soliton solutions. In the KIS regime, the soliton solution does not change from the single-pumped one except for a constant drift, which is associated with the time dependence of the reference pump field. Additionally, the reference pump field and the DKS field are stationary with respect to each other in the KIS regime. We can change the frame of reference in Eq. (3) to obtain an equation with stationary solutions:

$$\theta - \frac{(\alpha_{\text{ref}} - \alpha_{\text{pmp}} + \mathcal{D}(\mu_s))}{\mu_s} t = \theta', \quad t = t'. \quad (4)$$

This frame of reference rotates at an angular velocity where the reference pump is stationary. Using

$$\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \theta'}, \quad (5)$$

$$\frac{\partial}{\partial t} = -\frac{(\alpha_{\text{ref}} - \alpha_{\text{pmp}} + \mathcal{D}(\mu_s))}{\mu_s} \frac{\partial}{\partial \theta'} + \frac{\partial}{\partial t'}, \quad (6)$$

we can rewrite Eq. (3) as

$$\begin{aligned} \frac{\partial \psi}{\partial t} = & -(1 + i\alpha_{\text{pmp}})\psi + i \sum_{\mu} \mathcal{D}(\mu) \Psi e^{i\mu\theta} \\ & + \frac{(\alpha_{\text{ref}} - \alpha_{\text{pmp}} + \mathcal{D}(\mu_s))}{\mu_s} \frac{\partial \psi}{\partial \theta} + i|\psi|^2\psi \\ & + F_{\text{pmp}} + F_{\text{ref}} \exp[-i\mu_s\theta]. \end{aligned} \quad (7)$$

For convenience, we have dropped the primes in Eq. (7). The MLLE in the form of Eq. (7) admits stationary solutions in the KIS regime as the right-hand side is time-independent. Now, the stability of the soliton in the presence of perturbations can be studied using dynamical techniques. For the given set of parameters,

we can calculate the stationary solution ψ_0 using the Levenberg–Marquardt algorithm such that $\frac{\partial \psi_0}{\partial t} = 0$. Following the procedure described in Ref [46], we linearize Eq. (7) around ψ_0 to obtain

$$\frac{\partial \Delta \psi}{\partial t} = \mathcal{L}(\psi_0) \Delta \psi, \quad (8)$$

where $\Delta \psi$ is a perturbation, and $\mathcal{L}(\psi_0)$ is the linearized operator obtained numerically. The eigenvalues of this operator are referred to as the spectrum of the linearized operator or the dynamical spectrum, and they determine the local stability of the system [47,48]. By solving Eq. (8), we obtain the effect of the system on the perturbation after time Δt :

$$\Delta \psi(t_0 + \Delta t) = \exp[\mathcal{L}(\psi_0) \Delta t] \Delta \psi(t_0). \quad (9)$$

In general, one can decompose any perturbation into a linear combination of eigenfunctions v_n of $\mathcal{L}(\psi_0)$, with corresponding eigenvalues λ_n . Therefore,

$$\Delta \psi(t_0) = \sum_n a_n v_n, \quad \Delta \psi(t_0 + \Delta t) = \sum_n \exp[\lambda_n \Delta t] a_n v_n. \quad (10)$$

From Eq. (10), it is clear that a perturbation to the DKS grows exponentially if $\Re(\lambda_n) > 0$, persists if $\Re(\lambda_n) = 0$, and damps exponentially if $\Re(\lambda_n) < 0$. In all cases, there is one eigenvalue λ_{ps} , referred to as the position-shifting eigenvalue, whose eigenfunction corresponds to perturbations of the DKS position inside the cavity. Perturbations of the form of the position-shifting eigenfunction are responsible for fluctuations in the repetition rate of the soliton. For the singly pumped DKS [Fig. 3(a)], $\lambda_{ps} = 0$, which implies that perturbations to the soliton position due to noise persist. λ_{ps} in the singly pumped DKS exists at zero because the DKS has a degree of freedom associated with its translational invariance, which is due to the fact that the singly pumped microresonator system is perfectly symmetric in the resonator's azimuthal coordinate. However, in the KIS regime, the soliton is trapped by the two pumps and results in $\lambda_{ps} < 0$ [Figs. 3(b)–3(d)], which implies that perturbations to the DKS damp exponentially. In other words, the injection of the reference pump breaks the symmetry of the resonator, meaning that the DKS does not exhibit any translational invariance anymore. When the reference pump's frequency is at the center of the synchronization window, $\lambda_{ps} = -1$ [Fig. 3(d)], which corresponds to exponential damping of the DKS jitter with the photon lifetime. This is the most negative value that λ_{ps} can take as all photons in the cavity take on average a photon lifetime to exit, and therefore sets the fundamental limit for damping of the repetition rate noise, which after accounting for the normalization with respect to the total loss $\kappa = 280$ MHz corresponds to exponential damping of the intracavity noise by the photon lifetime $\tau_{\text{phot}} = 1/\kappa$. A physically intuitive picture as to why the photon lifetime is important can be obtained as follows. The repetition rate noise manifests as a perturbation to the soliton's intracavity mode frequencies. In the KIS regime, the soliton begins to converge to an equilibrium from this perturbed state. However, the photons that were a part of the perturbation have to exit the cavity in order for the system to be in equilibrium. Owing to the Q of the cavity, these photons leave the cavity on average in a photon lifetime. Therefore, a lower photon lifetime enables the noise to dampen faster. To verify the stability analysis conducted, we performed stochastic LLE simulations accounting for intracavity TRN, for which the repetition rate can be extracted at every round-trip time and processed to obtain its

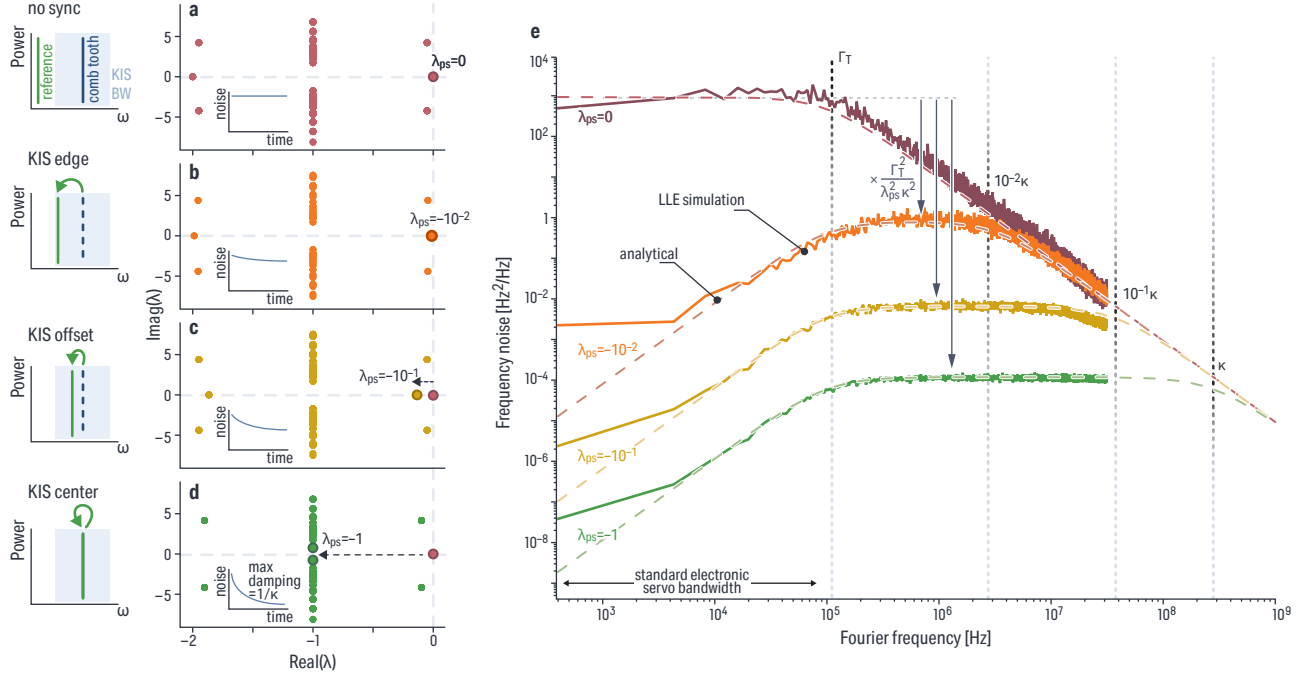


Fig. 3. Theoretical study of intrinsic noise suppression in KIS. (a) Eigenvalue spectrum of the linearized operator (dynamical spectrum) in the unsynchronized case. The highlighted eigenvalue (large red circle) is the position-shifting eigenvalue λ_{ps} , which in this case is null and corresponds to perturbations, like thermo-refractive noise (TRN) fluctuation of the DKS group velocity, persisting (inset). A similar study as in (a), with the reference at (b) the edge of the KIS window corresponding to $\lambda_{ps} = -10^{-2}$, (c) offset from the center of the KIS window at $\lambda_{ps} = -10^{-1}$, and (d) at the center of the KIS window, that is, in the original location of the soliton comb tooth, corresponding to the minimum value of $\lambda_{ps} = -1$. In each case, the noise is damped at a rate of $\lambda_{ps}\kappa$, where κ represents the photon loss rate (inset). (e) The power spectral density of the repetition rate noise, S_{rep} , is numerically simulated using the LLE (solid line) and is in accordance with the analytical analysis presented in Eq. (13) (dashed lines). In the unsynchronized case where $\lambda_{ps} = 0$ (red), the noise follows the pattern determined by the TRN, featuring a high-frequency cutoff at $\Gamma_T \ll \kappa$. When $\lambda_{ps} < 0$, indicating a synchronized case, the noise is damped at a rate of $\lambda_{ps}\kappa$. This leads to a significant reduction of the noise below Γ_T and forms a plateau for frequencies between Γ_T and $|\lambda_{ps}|\kappa$. This plateau exhibits a decrease from the single-pump plateau by a factor of $\Gamma_T^2/\lambda_{ps}^2\kappa^2$. For frequencies exceeding the KIS actuation bandwidth $|\lambda_{ps}|\kappa$, the noise follows the pattern of the single-pump TRN, which aligns well with the LLE simulation for $\lambda_{ps} = -10^{-2}$ (orange). However, for larger values of $|\lambda_{ps}|$, the LLE simulation becomes too computationally demanding to resolve at a high-frequency cutoff that is analytically confirmed. The maximum damping is achieved at the center of the KIS window ($\lambda_{ps} = -1$), with the widest KIS actuator bandwidth at κ (green).

PSD. These simulations account for the impact of TRN on pump detunings $\alpha_{p,r}$ and the dispersion $\mathcal{D}$ [21,30], and use either simulated or experimentally determined values for other parameters (see Supplement 1). In the unsynchronized case [Fig. 3(a)], the simulated repetition rate noise matches well with the analytical one that can be obtained following Ref. [30]:

$$S_{rep}^{1p}(f) = 2 \frac{\eta_T^2 k_B T^2}{\rho C V} \frac{\Gamma_T}{(f^2 + \Gamma_T^2)}, \quad (11)$$

with $\eta_T = \partial\omega_{rep}/\partial T|_{T_0}$ calculated when not synchronized (details in Supplement 1), ρ the material density, C its specific heat, V the modal volume of the fundamental transverse electric mode, Γ_T the thermal dissipation rate, T the temperature, k_B the Boltzmann constant, and f the Fourier frequency. Details of the parameter values and the methods by which they were obtained can be found in Supplement 1. Consistent with the linear stability analysis, the ω_{rep} PSD result from the stochastic LLE exhibits a much lower noise level in the KIS regime [Fig. 3(d)], which is minimized when the reference is at the center of the KIS bandwidth (i.e., where the comb tooth is originally). In our simulation, the frequency noise of the pumps has been neglected, yielding driving fields that are perfect Dirac delta functions spectrally and hence resulting in the reduction of the PSD at low Fourier frequencies. Interestingly, one can derive an analytical expression for the repetition rate noise in

the KIS regime, which can be obtained from the linear stability analysis of the MLE regime:

$$\frac{\partial \Delta\omega_{rep}}{\partial t} = \kappa \lambda_{ps} \Delta\omega_{rep} + \eta_T \frac{\partial \Delta T}{\partial t}, \quad (12)$$

with $\kappa = 1/\tau_{phot}$ the total loss rate of the cavity and τ_{phot} the photon lifetime, λ_{ps} the position-shifting eigenvalue, and $\Delta\omega_{rep}$ the repetition rate noise. The PSD of the repetition rate noise can then be derived following Supplement 1 such that

$$S_{rep}^{kis}(f) = S_{rep}^{1p}(f) \times \frac{f^2}{f^2 + \lambda_{ps}^2 \kappa^2}. \quad (13)$$

Outside of synchronization $\lambda_{ps} = 0$ yields the same expression as the single-pump case [Eq. (11)] with a typical Lorentzian profile in the Fourier frequency space. In the KIS regime, the noise spectrum is then damped by the $f^2/(f^2 + \lambda_{ps}^2 \kappa^2)$ term, linking KIS to the energy exchange rate of the system, as expected from attractors of nonlinear dissipative systems, and is in good agreement with the simulations [Fig. 3(d)], albeit with a discrepancy at low Fourier frequency due to the finite simulation time and averaging. In comparison to the conventional single-pump DKS, the PSD profile of $\Delta\omega_{rep}$ is dramatically altered, since at low frequency, it follows a $S_0 f^2 / \Gamma_T^2 \lambda_{ps}^2 \kappa^2$ profile, with $S_0 = 2 \frac{\partial\omega_{rep}}{\partial T} \frac{k_B T^2 \Gamma_T}{\rho C V}$. As the

thermal dissipation rate is much slower than the photon decay rate ($\Gamma_T \ll \kappa$) in the microring, this hints at the good long-term stability of the repetition rate, which will no longer be hindered by incoherent intracavity noise processes, such as TRN, in contrast to the single-pump case that exhibits a typical plateau at S_0/Γ_T^2 at low Fourier frequencies. At high Fourier frequency $f > |\lambda_{ps}|\kappa$, the PSD in KIS will follow the Lorentzian profile imposed by the thermal noise, yet the Fourier frequency at which it happens is λ_{ps} dependent since a plateau at $S_0/\lambda_{ps}^2\kappa^2$ will be observed for $f \in [\Gamma_T; |\lambda_{ps}|\kappa]$; however, this plateau occurs at a value that is still much lower than the noise in the single-pump case, which follows a $1/f^2$ trend, and is reduced from the single-pump low-frequency plateau by a factor $\Gamma_T^2/\lambda_{ps}^2\kappa^2$. Physically, it is important to note that the statistical fluctuations of the refractive index of the material leading to TRN remain present in the microresonator; however, KIS provides a trapping of the DKS. The dynamics of attractors in nonlinear dissipative systems tells us that any perturbations faster than their characteristic energy exchange rate cannot be counteracted. In KIS, the exchange rate is defined by the photon lifetime $\tau_{\text{phot}} = 1/\kappa$ and the reference laser frequency relative to the KIS bandwidth fixing the zero mode eigenvalues λ_{ps} ; any noise faster than $|\lambda_{ps}|\kappa$ will not be influenced by the soliton synchronization and will be experienced as in the single-pumped regime. The dependence on λ_{ps} can be understood such that at the edge of the KIS bandwidth, the synchronization is slower since the soliton must adapt its repetition rate within the nonlinear Kerr effect timescale. This results in a small λ_{ps} , and hence, $S_{\text{rep}}^{\text{KIS}}$ catches up with $S_{\text{rep}}^{\text{LP}}$ at relatively low frequencies. In contrast, at the center of the KIS bandwidth, synchronization is faster since the reference pump is already at the frequency for which the soliton exists in the single-pump case. This results in a larger $|\lambda_{ps}|$ and a higher frequency at which $S_{\text{rep}}^{\text{KIS}}$ catches up with $S_{\text{rep}}^{\text{LP}}$. The important aspect of KIS is that the system noise is not limited by material property nor cavity volume but instead becomes photon-lifetime-limited. However, it limits both the loading time for which the reference can be injected into the microring resonator and the photon dissipation of the previously unsynchronized cavity soliton. Importantly, the KIS window scales with the geometric mean of the reference power and comb tooth power. Thus, RIN can affect the reference frequency position within the KIS window. However, simulations (Supplement 1) show that λ_{ps} clamps to -1 over a wide frequency range inside the KIS window, indicating maximum KIS intracavity noise damping is insensitive to amplitude noise.

The above results are similar to the so-called “quantum diffusion limited” counterpropagative solitons [49]. Here, the theoretical timing jitter limitation of solitons is obtained through a Lagrangian approach [50], where an equivalent to KIS occurs between solitons in each direction [51] since both systems obey an analogous Alder equation. This analysis yields a noise limited by the photon lifetime $1/\kappa$. Importantly, here we expand such a quantum diffusion limit to a single soliton, where the synchronization is enabled through entirely controllable external parameters provided by the two laser pumps, instead of using synchronization between two soliton states. We note that while ultra-high Q provides net benefits in terms of reducing the pump power needed to generate Kerr solitons, when it comes to minimizing the impact of intrinsic noise, it is advantageous to reduce Q to have a photon lifetime $1/\kappa$ as short as possible. In this context, Si_3N_4 microring resonators present a critical advantage under KIS compared to much larger volume crystalline resonators with ultra-high Q (e.g., 10^9) [52],

since our microring exhibits a decrease in the photon lifetime by about three orders of magnitude while retaining a small enough modal volume to enable relatively low-pump-power DKS existence, and no longer suffers TRN-related limitations thanks to KIS. Importantly, our conclusions provide precision to the current state of understanding of KIS-like systems. Specifically, it has been hypothesized that the synchronized soliton may remain susceptible to TRN [34], in a similar fashion to single-pumped DKS. Using this understanding, to obtain a sub-TRN DKS repetition rate noise, the cavity TRN shall be reduced through, for instance, anti-correlated main and reference pumps that counteract the TRN effects on the cavity; such an argument has been brought forward to explain below-TRN synchronized-DKS repetition rate noise [53]. In sharp contrast, our theoretical demonstration shows that the soliton adapts its group velocity to compensate for refractive index variations due to TRN, maintaining a fixed repetition rate dictated by the KIS dual-pinning of the comb within the actuation bandwidth allowed by the soliton lifetime. Therefore, even if anti-correlated main and reference pumps reduce the cavity TRN, our results show that this dual-pinning fundamentally changes the dynamics of the DKS and decouples it from the intracavity noise for Fourier frequencies below the inverse of the photon lifetime. Thus, the KIS dual-pinning does not require any common coherence between the pumps for sub-TRN DKS noise, which we aim to also demonstrate experimentally.

4. EXPERIMENTAL DEMONSTRATION USING OCTAVE-SPANNING SELF-REFERENCED COMB

Finally, we experimentally demonstrate that KIS quenches the intrinsic noise of the repetition rate (and thus of the individual comb lines), which is now entirely determined by the frequency noise and spectral separation between the two pinned teeth regardless of whether the actuation of the thermo-refractive index by the main and reference pump is coherent. This can be demonstrated by leaving the main and reference pump lasers to be free-running. In the fully free-running lasers case, the two pinned teeth correspond to the main and reference pumps, as previously demonstrated [28,34], thereby yielding optical frequency division (OFD) by a factor μ_r^2 , and the two lasers do not show any common coherence nor (anti)-correlation of their respective actuation of the thermo-refractive effect onto the microring resonator. One can verify the OFD against the same system when the two lasers become correlated by locking the CEO through the main pump (i.e., self-referencing), which in this case sees its frequency variation becoming $\Delta\omega_{\text{pmp}} = \Delta\omega_{\text{ref}} \times (M_{\text{ref}} - \mu_r)/M_{\text{ref}}$ with $\Delta\omega_{\text{ref}}$ the variation of frequency of the reference pump, and the OFD becomes $M_{\text{ref}}^2 = [(\omega_{\text{ref}} - \omega_{\text{ceo}})/\omega_{\text{rep}}]^2$. Experimentally, we use the same microring resonator as in Fig. 2(c), pumped at $\omega_{\text{pmp}}/2\pi = 285.467836 \text{ THz} \pm 3 \text{ MHz}$ ($\approx 1050.8 \text{ nm}$) and creating a DW at which we send the reference pump at $\omega_{\text{ref}}/2\pi = 192.559616 \text{ THz} \pm 2 \text{ MHz}$ ($\approx 1557.4 \text{ nm}$), with both frequencies obtained after measuring the repetition rate and CEO as we describe later. The uncertainty is a one standard deviation value that is dominated by the repetition rate uncertainty. Frequency doubling the reference pump using a periodically poled lithium niobate waveguide enables $f - 2f$ interferometry against the short DW at $385.369148 \text{ THz} \pm 4 \text{ MHz}$ ($\approx 778.4 \text{ nm}$) for CEO detection [Fig. 4(a)]. The CEO can be efficiently detected with more than 60 dB dynamic range as a consequence of the reference pump laser being a comb tooth that provides a large power

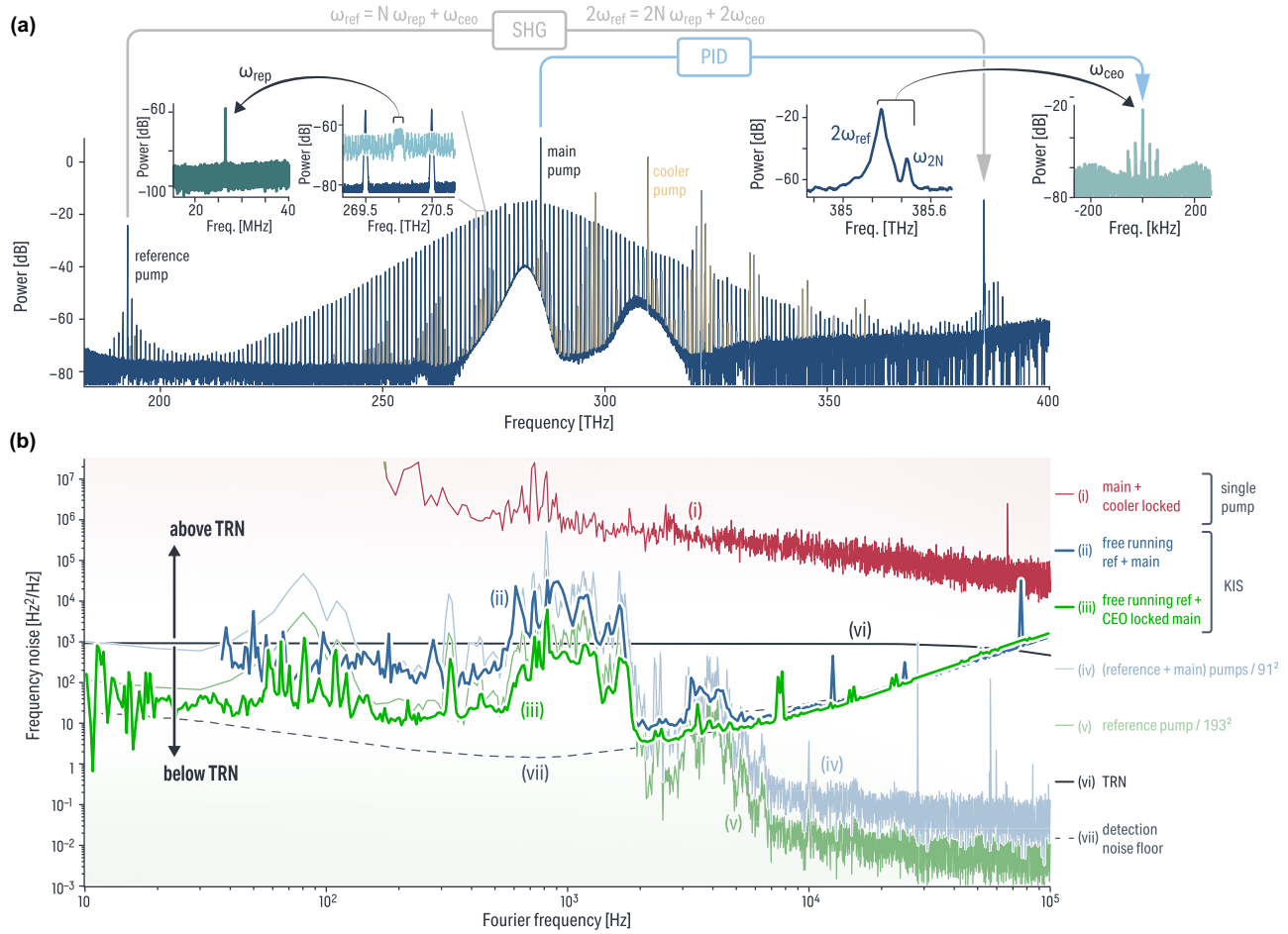


Fig. 4. Thermo-refractive noise quenching in an octave-spanning microcomb: free-running and self-referenced. (a) Experimental spectrum of the octave-spanning microcomb pumped at $\omega_{\text{pmp}}/2\pi \approx 285.5$ THz with the reference pump at $\omega_{\text{ref}}/2\pi \approx 192.6$ THz. A strong cross-polarized cooler pump is used to thermally stabilize the resonator (yellow spectrum). The frequency-doubled reference is obtained through a second-harmonic generation (SHG) nonlinear crystal and beat against the closest comb tooth at $2f$. The repetition rate is measured around 270 THz with an electro-optic comb enabling detection within a 50 MHz bandwidth. (b) Frequency noise obtained for a single-pumped microcomb with frequency-locked cooler (i, red), KIS microcomb with free-running main and reference pumps (ii, dark blue), and KIS microcomb with free-running reference pump and locked CEO through feedback to the main pump (iii, green). The frequency noise of the pumps, which is similar for both reference and main pump, is overlaid in thin lines for each KIS case after accounting for the relevant optical frequency division (OFD) factor (iv)–(v). When both pumps are free-running, the OFD = 91^2 corresponds to the comb tooth separation between the pumps (iv), and accounting for both pumps not sharing any common noise is in good agreement with the detected repetition rate noise. In the CEO-locked case (v), OFD = 193^2 , which further reduces the repetition rate noise (here the noise on the CEO lock can be neglected, and only the free-running reference pump noise is a factor). In both KIS cases, the single-pumped (without cooler) TRN limit (vi, black solid line) is beaten. The detection noise floor (vii, dashed line) is limited by the RF synthesizer used in the electro-optic comb repetition rate measurement apparatus.

for doubling and the increase in short DW power through the self-balancing effect in KIS [28,54]. We use electro-optic modulation to detect $\omega_{\text{ceo}}/2\pi = -249.91559097$ GHz ± 10 Hz (from electrical spectrum analyzer recording bandwidth) and phase-lock it to a 10 MHz rubidium frequency standard, by actuating the main pump frequency through a proportional–integral–derivative (PID) controller [Fig. 4(a)]. In order to measure the repetition rate $\omega_{\text{rep}}/2\pi = 999.01312$ GHz (± 10 kHz from its linewidth under free-running operation [28]), we use a similar electro-optic apparatus that modulates two adjacent Kerr comb teeth. This results in their spectral translation through higher-order modulation sidebands, creating a beat note between a sideband from each Kerr comb tooth that resides within a 50 MHz bandwidth and is detected by a low-noise avalanche photodiode. We proceed to measure S_{rep} for three different cases: single pump [Fig. 4(b)(i)],

KIS with free-running pumps [Fig. 4(b)(ii)], and KIS with a free-running reference and locked CEO [Fig. 4(b)(iii)]. In the first case, we observe the $1/f^2$ trend that is a signature of the TRN-limited behavior expected for a single-pumped cavity soliton, as presented above. It is worth pointing out that we use a cooler pump (counterpropagating and cross-polarized relative to the main pump) to thermally stabilize the cavity and provide adiabatic access to the DKS. While a low-power cooler (in the 10 mW range) can suppress TRN in microring resonators by about an order of magnitude [21], we do not observe such effects. In particular, the simulated S_{rep}^{1p} without cooler is orders of magnitude lower than our observed noise spectrum [Fig. 4(b)(vi)]. We note that for soliton access in our system, the cooler pump must be sufficiently strong, here between 150 and 300 mW of on-chip power [55,56], to counteract the thermal shift induced by the 150 mW power main pump and

access the DKS state [57,58]. Theoretically, it has been shown that thermal squeezing of integrated cavities works efficiently only for a low-power cooler, while actually deteriorating the TRN once a critical power threshold is passed [29]. Therefore, the use of a pump cooler to generate a DKS, although convenient, can greatly degrade its noise metrics when not synchronized.

In the KIS regime, $S_{\text{rep}}^{\text{KIS}}$ does not follow the TRN profile, as expected from theory, as the frequency noise PSD at 2.5 kHz is brought down from $S_{\text{rep}}^{\text{LP}} \approx 4 \times 10^5 \text{ Hz}^2/\text{Hz}$ to $S_{\text{rep}}^{\text{KIS}} \approx 10 \text{ Hz}^2/\text{Hz}$ in the completely free-running KIS case [Fig. 4(b)(ii)]. When comparing $S_{\text{rep}}^{\text{KIS}}$ and the expected TRN-limited PSD in the single-pump case (without the cooler), one can observe that the KIS microcomb has already bypassed the otherwise intrinsic noise limitation of the cavity soliton, beating TRN. We also demonstrate that $S_{\text{rep}}^{\text{KIS}}$ is determined by the OFD factor of the KIS operation. In the above case of two free-running pumps, the measured repetition rate noise is well-reproduced by considering the combined pumps' noise and $\text{OFD} = \mu_s^2 = 91^2$ [Fig. 4(b)(iv)]. This demonstrates that the soliton's response to the intracavity fluctuation of the microring refractive index is damped regardless of the two lasers' impact on the thermo-refractive shifts of the microring resonances and is only limited by the phase noise of the two pumps, which in the free-running case is dominant. When the CEO is locked through actuation of the main pump frequency, $S_{\text{rep}}^{\text{KIS}}$ is further reduced thanks to the larger $\text{OFD} = M_{\text{ref}}^2 = 193^2$ [Fig. 4(b)(v)] and is in good agreement with the measured reference pump frequency noise divided by this larger OFD factor. In this scenario, the reference remains free-running, while TRN is further beaten by a factor of 15 at low frequency. At high frequency, $S_{\text{rep}}^{\text{KIS}}$ is limited by the detection floor of the electro-optic apparatus used to measure the $\approx 1 \text{ THz}$ repetition rate of the KIS DKS [Fig. 4(b)(vii)]. Since our system is limited by the pump(s) frequency noise in both cases, KIS-damped TRN does not constrain the repetition rate performances. We set our reference to be near the KIS window center where $\lambda_{ps} \approx -1$. Achieving larger λ_{ps} values, where KIS internal damping would exceed our noise floor, would place the system too close to the KIS edge, leading to stability issues—an investigation beyond this manuscript's scope.

5. CONCLUSION

In conclusion, we have demonstrated both theoretically and experimentally that Kerr-induced synchronization (KIS) enables, in an all-optical fashion, quenching of internal noise sources in integrated optical frequency combs at a magnitude and with a bandwidth determined by the photon lifetime, thereby making the experimental comb noise metrics mostly dependent on the noise properties of the two pumps, regardless of their shared properties. Thanks to the phase locking of the cavity soliton onto the reference laser, the comb exhibits a dual-pinning behavior that modifies the so-called elastic-tape model describing the noise propagation across comb teeth. This enables the effective linewidths of the comb teeth to remain within the same order of magnitude as the pump lasers, even for mode separations down to the carrier envelope offset (CEO), in contrast to the single-pump soliton case where the comb tooth linewidth can be hundreds of times the pump linewidth at the edges of the comb spectrum, which particularly impacts the linewidth of the CEO. Importantly, the simple frequency noise propagation from tooth to tooth accurately matches the KIS comb and CEO linewidth experiment, while

the single-pump model diverges from the experiment. This is explained by the importance of thermo-refractive noise (TRN) in the single-pump case, which in contrast is relatively unimportant due to its quenching in the KIS case. To study the impact of the TRN on the microcomb properties, we demonstrate theoretically through a linear stability analysis of the Lugiato–Lefever equation that KIS damps the intracavity noise fluctuations in accordance with the dynamics of attractors in nonlinear damped systems, where noise reduction is determined by the energy exchange rate of the system, here the total loss rate κ . This holds despite the injection of another laser into the microring resonator, which could potentially raise its temperature and degrade the statistical fluctuations of the cavity refractive index. Hence, regardless of the thermo-optic actuation of the main and the reference pump and their respective correlations, the soliton self-adapts its repetition rate to compensate for the variation of refractive index inside the microcomb to follow the dual-pinning condition. We show that the maximum damping is at the center of the synchronization bandwidth (i.e., where the comb tooth was previously located in a single-pump regime). This is confirmed experimentally by measuring the repetition rate noise of the microcomb with free-running lasers, which is found to be entirely determined by the independent pump laser frequency noise and the optical frequency division (OFD) factor from the dual-pinning spacing, while importantly becoming independent of the thermo-refractive noise (TRN) of the cavity. The repetition rate noise is suppressed by more than four orders of magnitude compared to free-running single-pumped DKS and is about an order of magnitude lower than the minimum TRN that the singly pumped DKS could exhibit in our system. Such a demonstration is reproduced when the two lasers become correlated through self-referencing of the microcomb by measuring and locking the CEO by feedback to the main pump. Here, the microcomb repetition rate noise is further decreased (by a factor of 15) due to the larger OFD, and once again is only determined by the frequency noise of the reference laser and not the intracavity noise. Our work demonstrates that despite the low modal volume of integrated microcombs, which makes them predisposed to large TRN that can greatly hinder their performance, KIS can enable ultra-low-noise performance since the repetition rate noise becomes dependent only on the extrinsic noise properties of the pump lasers and the cavity photon lifetime. This dramatic noise reduction is critically important for the octave-spanning combs used in self-referencing, which thus far has only been achieved in microresonators at high repetition rates (small cavity volumes). Optimization of the photon lifetime can be engineered by leveraging the coupling dispersion of pulley bus–ring couplers [59], further lowering the noise floor of the KIS microcomb. The external driving noise can be mitigated, as laser locking using on-chip reference cavities has been shown to yield low-noise performance [60]. Hence, the noise performance of a laser that is locked to a large cavity can be transferred to the smaller microcomb cavity, with the latter no longer a limitation due to KIS, thereby providing a pathway for TRN transfer. Our work shows that KIS is a promising approach for the development of low-noise, self-stabilized integrated microcombs with a low footprint for SWaP optimization and performance that can potentially offer on-par performance with state-of-the-art fiber-based optical frequency combs.

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Grégory Moille designed the resonator, performed the experimental work, and led the project. Pradyoth Shandilya developed the theoretical understanding and the analytical framework and performed the simulation. Jordan Stone provided help in theoretical and experimental noise analysis and understanding. Kartik Srinivasan and Curtis Menyuk provided guidance, feedback, and secured funding. Grégory Moille, Pradyoth Shandilya, and Kartik Srinivasan wrote the manuscript, with input from all authors. All the authors contributed to and discussed the content of this manuscript.

Disclosures. Grégory Moille, Curtis Menyuk, and Kartik Srinivasan have submitted a provisional patent application based on aspects of the work presented in this paper.

Data availability. The data that support the plots within this paper and other findings of this study are available from the corresponding authors upon request.

Supplemental document. See Supplement 1 for supporting content.

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All-optical noise quenching of an integrated frequency comb: supplement

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Supplementary Information: All-Optical Noise Quenching of An Integrated Frequency Comb

S.1 Derivation of analytical repetition rate noise

Temperature fluctuations in a microring resonator can be described by a Langevin equation,

$$\frac{d\Delta T}{dt} = -\Gamma_T \Delta T + \zeta_T, \quad (\text{S.1.1})$$

where Γ_T is the thermal dissipation rate and ζ_T is a white-noise fluctuation source defined by its autocorrelation,

$$\langle \zeta_T(t) \zeta_T(t + \tau) \rangle = \frac{\Gamma_T k_B T^2}{\rho C V} \delta(\tau). \quad (\text{S.1.2})$$

with the parameters as defined in the main text after Eq. (11). Taking the Fourier transform of Eq. (S.1.1), we get:

$$-if \Delta \tilde{T}(\nu) = -\Gamma_T \Delta \tilde{T}(\nu) + W(\nu). \quad (\text{S.1.3})$$

Therefore,

$$\Delta \tilde{T}(\nu) = \frac{W(\nu)}{\Gamma_T - i\nu} \quad (\text{S.1.4})$$

$$S_{\tilde{T}}(\nu) = \frac{k_B T^2}{\rho C V} \frac{2\Gamma_T}{\Gamma_T^2 + \nu^2} \quad (\text{S.1.5})$$

For small fluctuations in temperature, the change in repetition rate can be written as:

$$\Delta \omega_{\text{rep}} = \eta_T \times \Delta T \quad (\text{S.1.6})$$

with $\eta_T = \left. \frac{\partial \omega_{\text{rep}}}{\partial T} \right|_{T_0}$ the variation of the repetition rate with temperature around T_0 , with more detail provided in Supplementary Information S.3. From Eq. (S.1.6), we can determine:

$$\Delta \tilde{\omega}_{\text{rep}}(\nu) = \eta_T \frac{W(\nu)}{\Gamma_T - i\nu} \quad (\text{S.1.7})$$

Hence, when the DKS is under single pump operation (*i.e.* not synchronized to the reference), the power spectral density (PSD) of the frequency noise is given by:

$$S_{\omega_{\text{rep}}}(\nu) = 2 \frac{\eta_T^2 k_B T^2}{\rho C V} \frac{\Gamma_T}{\Gamma_T^2 + \nu^2} \quad (\text{S.1.8})$$

In the Kerr-induced synchronization (KIS) regime, from the linearization of the LLE presented in the main body of the manuscript, which yields the eigenvalues λ , one can define another Langevin equation that can be written assuming that fluctuations to the repetition rate damp exponentially,

$$\frac{\partial \Delta \omega_{\text{rep}}}{\partial t} = \lambda_{ps} \kappa \Delta \omega_{\text{rep}} + \eta_T \frac{d\Delta T}{dt} \quad (\text{S.1.9})$$

where $\kappa = 1/\tau_{\text{phot}}$ is the total loss rate accounting for the microresonator internal loss (from scattering and absorption) and the coupling loss, and is inversely proportion to the photon lifetime (τ_{phot}). Repeating the same operation as in Eq. (S.1.3):

$$\Delta \tilde{\omega}_{\text{rep}}(\nu) = \eta_T \frac{ifW(\nu)}{(\Gamma_T - i\nu)(\lambda_{ps}\kappa - i\nu)} \quad (\text{S.1.10})$$

yielding:

$$S_{\omega_{\text{rep}}}(\nu) = 2 \frac{\eta_T^2 k_B T^2}{\rho C V} \frac{\Gamma_T \nu^2}{(\Gamma_T^2 + \nu^2)(\lambda_{ps}^2 \kappa^2 + \nu^2)} \quad (\text{S.1.11})$$

First, it is interesting to note that to minimize $S_{\omega_{\text{rep}}}(\nu)$, one needs to minimize λ_{ps} , which corresponds well with the LLE simulation occurring at the center of the KIS bandwidth, where $\lambda_{ps} = -1$. Secondly, outside of synchronization we obtained $\lambda_{ps} = 0$, which correctly makes Eq. (S.1.10) equal Eq. (S.1.8).

The physical quantities used in simulations, which match the ring resonator design that yields the microcombs in Figures 2 and 4 are summarized in Table S.II.

S.2 Multi-driven Lugiato-Lefever equation details

Here we detail the normalization used to arrive at the Lugiato-Lefever equation (LLE) in Eq. (3) of the main text. We assume that one can readily arrive at the LLE from the nonlinear Schrödinger equation with periodic boundary condition at every round trip¹, leading to:

$$\frac{\partial a(\theta, t)}{\partial t} = -\left(\frac{\kappa}{2} + i\delta\omega_{\text{pmp}}\right)a - i\gamma L|a|^2a + i\frac{D_2}{2}\frac{\partial^2}{\partial\theta^2}a + \sqrt{\kappa_{\text{ext}}P_{\text{p}}} \quad (\text{S.2.1})$$

with a the intracavity field, θ the azimuthal coordinate of the resonator moving at the speed of the DKS such that $\theta \mapsto \theta - \omega_{\text{rep}}t$, $\delta\omega_{\text{pmp}}$ the pump detuning relative to the cold resonance frequency, $\kappa = 2\kappa_{\text{ext}}$ the total loss, which in the critical coupling case equals twice the external (coupling) losses, $\gamma = 3.1 \text{ W}^{-1}\cdot\text{m}^{-1}$ the effective nonlinear coefficient of the system at the 285.5 THz pump frequency (calculated through the finite-element method (FEM)), $L = 2\pi R = 144.51 \text{ }\mu\text{m}$ the perimeter of the resonator, and D_2 the group velocity dispersion of the system.

One could extend the previous equation in the case of higher-order dispersion coefficients such that $D_{\text{int}}(\mu) = \sum_{k>1} \frac{D_k}{k!} \mu^k$, with μ the azimuthal mode relative to the pump, while noticing that $\sum_{k>1} (-i)^{k+1} \frac{D_k}{k!} \frac{\partial^k}{\partial\theta^k} a = \sum_{\mu} D_{\text{int}}(\mu) A(\mu, t) \exp(i\mu\theta)$, where $A(\mu) = \text{FT}[a(\theta, t)]$ is the Fourier transform of the intracavity field from θ to μ .

In addition, a secondary pump can be introduced into the system following ref 2 and leading to:

$$\begin{aligned} \frac{\partial a(\theta, t)}{\partial t} = & -\left(\frac{\kappa}{2} + i\delta\omega_{\text{pmp}}\right)a - i\gamma L|a|^2a + \sum_{\mu} D_{\text{int}}(\mu) A(\mu, t) e^{i\mu\theta} \\ & + \sqrt{\kappa_{\text{ext}}P_{\text{p}}} + \sqrt{\kappa_{\text{ext}}P_{\text{ref}}} \exp\left[i(\delta\omega_{\text{ref}} - \delta\omega_{\text{pmp}} + D_{\text{int}}(\mu_s))t - i\mu_s\theta\right] \end{aligned} \quad (\text{S.2.2})$$

with $\delta\omega_{\text{ref}}$ the detuning of the reference relative to its cold cavity, which is offset from the fixed frequency grid spaced by ω_{rep} by $D_{\text{int}}(\mu_s)$, with μ_s the mode at which the reference is injected in the cavity and where the synchronization will happen, and P_{ref} the reference power in the waveguide.

Using the change of variable $t \mapsto \frac{\kappa}{2}t$ we obtain Eq. (3) of the main text assuming the following normalization in Table S.I:

Table S.I: Normalized parameters in Eq. (3) of the main text

Parameter	Normalization
ψ	$\sqrt{\frac{2\gamma L}{\kappa}} a^*$
α_{p}	$-\frac{2}{\kappa} \delta\omega_{\text{pmp}}$
α_{ref}	$-\frac{2}{\kappa} \delta\omega_{\text{ref}}$
$\mathcal{D}(\mu)$	$-\frac{2D_{\text{int}}(\mu)}{\kappa}$
F_{pmp}	$\sqrt{\frac{2\kappa_{\text{ext}}\gamma L}{\kappa}} P_{\text{p}}$
F_{ref}	$\sqrt{\frac{2\kappa_{\text{ext}}\gamma L}{\kappa}} P_{\text{ref}}$

S.3 Details on the simulation of the thermo-refractive noise

Here, we describe the simulation details used to calculate the repetition-rate noise due to TRN using the LLE. The intracavity temperature is modeled as a stochastic variable with a white Gaussian driving force and subject to fluctuation dissipation. This yields the Langevin equation:

$$\frac{dT}{dt} = -\Gamma_T \Delta T + \zeta_T, \quad (\text{S.3.1})$$

where Γ_T is the thermal dissipation rate, and the temperature driving term ζ_T is defined by the autocorrelation function,

$$\langle \zeta_T(t) \zeta_T(t + \tau) \rangle = \frac{2\Gamma_T k_B T^2}{\rho CV}. \quad (\text{S.3.2})$$

Fluctuations in the intracavity temperature lead to fluctuations in the pump detunings,

$$\alpha_{\text{pmp}}(T) = -\frac{2}{\kappa} [\omega_{\mu_0}(T) - \omega_{\text{pmp}}], \quad (\text{S.3.3})$$

$$\alpha_{\text{ref}}(T) = -\frac{2}{\kappa} [\omega_{\mu_s}(T) - \omega_s], \quad (\text{S.3.4})$$

and in the dispersion relation,

$$\omega_{\mu}(T) = \omega_{\mu}(T_0) - (T - T_0) \times \eta_{\nu} \frac{\omega_{\mu}^2(T_0) L_{\text{RT}}}{(\mu + m)c}, \quad (\text{S.3.5})$$

where η_{ν} is the material thermo-optic coefficient, L_{RT} is the cavity round-trip length, m is the mode number of the pumped mode, and c is the speed of light in vacuum. The temperature dependent detunings and dispersion relation can be combined into the following expression for the effective integrated dispersion at temperature T ,

$$\mathcal{D}(\mu, T) = -\frac{2}{\kappa} [\omega_{\mu}(T) - \omega_{\mu_0}(T_0) - \mu D_1], \quad (\text{S.3.6})$$

where $D_1/2\pi$ is the microresonator free spectral range. This leads to the temperature dependent LLE being written as,

$$\begin{aligned} \frac{\partial \psi}{\partial t} = & -[1 + i\alpha_{\text{pmp}}(T)]\psi + i \sum_{\mu} \mathcal{D}(\mu, T) \tilde{\Psi} e^{i\mu\theta} + i|\psi|^2\psi \\ & + F_{\text{pmp}} + F_{\text{ref}} \exp\left[i\left(\alpha_{\text{ref}}(T) - \alpha_{\text{pmp}}(T) + \mathcal{D}(\mu_s, T)\right)t - i\mu_s\theta\right] \end{aligned} \quad (\text{S.3.7})$$

The change in repetition rate $\Delta\omega_{\text{rep}}$ due to change in temperature can be calculated directly from the temperature-dependent LLE by tracking the change in the soliton position over time

$$\Delta\omega_{\text{rep}}[i] = \frac{\theta_{\text{max}}[i] - \theta_{\text{max}}[i-1]}{\Delta t \times R}, \quad (\text{S.3.8})$$

where $[i]$ denotes the i^{th} time step. Monte-Carlo simulations were performed to calculate the PSD. The $\Delta\omega_{\text{rep}}$ data was recorded with a sampling frequency of 10^7 Hz and a frequency resolution of 62.3 Hz. The PSD was calculated by taking the discrete Fourier transform as $|\Delta\tilde{\omega}_{\text{rep}}(f)|^2$. To obtain a good estimate, 144 parallel simulations were performed at the UMBC HPCF and an averaged PSD was obtained. The quantity η_T that is used in Supplementary Information S.1 was found by calculating the change in repetition rate locally around the mean temperature T_0 using the temperature-dependent LLE as,

$$\eta_T = \left. \frac{\partial \omega_{\text{rep}}}{\partial T} \right|_{T_0} = \frac{\omega_{\text{rep}}(T_0 + \delta T) - \omega_{\text{rep}}(T_0 - \delta T)}{2\delta T}, \quad (\text{S.3.9})$$

for a small δT . The parameters used for the TRN simulation are summarized in Table S.II

S.4 Summary of physical parameters

The physical parameters used in this study to investigate the noise of the microcomb under KIS can be summarized in Table S.II below:

Table S.II: Parameter used for the design of interest considered to study the noise of the microcomb

Parameter	Values	Method
k_B	$1.38 \times 10^{-23} \text{ J}\cdot\text{K}^{-1}$	physical constant
R	$23 \text{ }\mu\text{m}$	design
H	670 nm	design
RW	850 nm	design
ρ	$3000 \text{ kg}\cdot\text{m}^{-3}$	refs 3,4
C	$700 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$	refs 3,4
V	$80 \text{ }\mu\text{m}^3$	FEM simulations
Γ_T	$1 \times 10^5 \text{ Hz}$	FEM simulations
η_ν	$2.45 \times 10^{-5} \text{ K}^{-1}$	ref 5
κ	$\approx 280 \times 10^6 \text{ Hz}$	Measurements
η_T	$42 \times 10^6 \text{ Hz}\cdot\text{K}^{-1}$	LLE simulations

S.5 Experimental setup

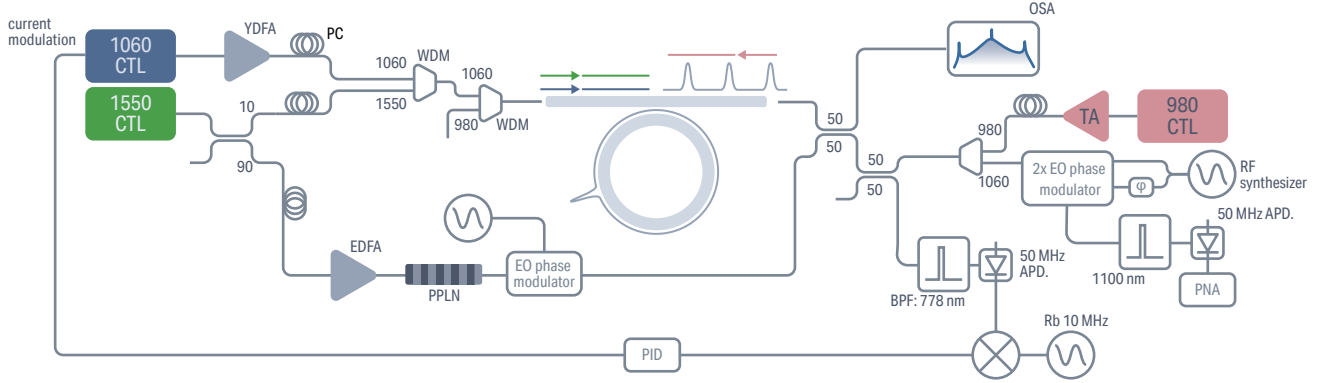


Fig. S.1 – Experimental setup for the comb generation, repetition rate noise and frequency noise analysis, and carrier envelope offset detection and locking. CTL: continuously tunable laser, YDFA: Ytterbium doped fiber amplifier, EDFA: erbium doped fiber amplifier, WDM: wavelength demultiplexer, PC: polarization controller, TA: taper amplifier, OSA: optical spectrum analyzer, EO: electro-optics, APD: avalanche photodiode, PNA: phase noise analyzer, PPLN: periodically polled lithium niobate, PID: proportional-integral-derivative controller, BPF: band-pass filter.

The experimental setup is depicted in Fig. S.1. We use a 283.5 THz main pump provided by a continuously tunable laser (CTL) that is amplified using a ytterbium-doped fiber amplifier (YDFA) to provide 150 mW of on-chip power. This on-chip power accounts for a wavelength-demultiplexer (WDM) and chip insertion losses of about 2 dB per facet when set in the transverse electric (TE) polarization through a polarization controller (PC) to generate the octave-spanning comb spectrum designed for this particular mode. The reference power is provided by another CTL, which is combined with the main pump using a WDM and sent to the chips with the same lensed fiber and also set in TE polarization. The cooler pump, which enables adiabatic tuning of the main pump onto the DKS step, is provided by a third CTL that is amplified using a tapered amplifier (TA) to provide 300 mW of on-chip power at 308.7 THz in a counter-propagating and cross-polarized fashion from the main pump. To do so, we use another pair of 980/1060 WDMs at the input/output to separate the main and cooler pump. In addition a 90/10 coupler is used at the output to process the optical frequency comb without artefacts from the transfer function of the WDM. From there, we use 50 % of the output signal to probe the comb spectrum with the optical spectrum analyzer (OSA), while the other 50 % is sent to the electro-optic (EO) comb apparatus, consisting of cascaded electro-optic phase modulators, to spectrally translate two adjacent Kerr comb teeth, to be within a 50 MHz separation from each other. This electrical beat note detected by an avalanche photodiode (APD) is then processed with a phase noise analyzer (PNA). For the carrier-envelope offset detection and stabilization, we use another level of 50/50 coupler to tap the high frequency dispersive wave. By sending the frequency doubled reference pump (obtained using a periodically polled lithium niobate (PPLN) crystal) to another EO phase modulator, the large CEO frequency can be measured through interference of the resulting signal with the tapped off high frequency dispersive wave, with a beat note in the 50 MHz band that

is detected with another APD. The signal is then locked using a PID controller to feed back onto the main pump frequency and lock the CEO-translated beat to a 10 MHz Rubidium frequency standard.

S.6 Comb linewidth measurement

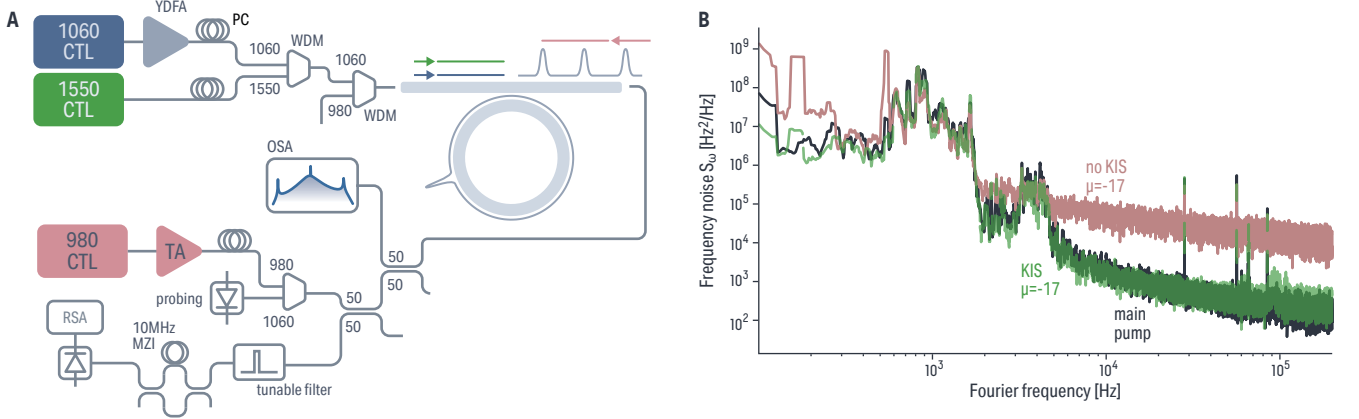


Fig. S.2 – (A) Experimental setup for the comb generation and linewidth analysis of individual comb teeth. CTL: continuously tunable laser, YDFA: Ytterbium doped fiber amplifier, EDFA: erbium doped fiber amplifier, WDM: wavelength demultiplexer, PC: polarization controller, TA: tapered amplifier, OSA: optical spectrum analyzer, RSA: real time spectrum analyser. (B) Frequency noise power spectral density of the main pump (black) and the comb line at $\mu = -17$ out of synchronization (red) and in KIS (green), exhibiting a clear change of the noise profile, from which the effective linewidth is extracted.

The comb linewidth measurement setup is depicted in Fig. S.2A. The main, reference, and cooler pumps are prepared in the same fashion as in Fig. S.1. On the output side, however, we use a Gaussian-shape band-pass fiber Bragg grating filter with 0.1 nm full-width half-maximum bandwidth and tunable center wavelength from 980 nm to 1140 nm, which enables us to filter out only one comb line of interest and send it to the 10 MHz unbalanced MZI optical frequency discriminator providing, after processing, the frequency noise of the μ^{th} tooth [Fig. S.2B]. From the frequency noise PSD $S_\omega(f)$, we extract the effective linewidth $\Delta\nu_{\text{eff}}$ such that:

$$\int_{\Delta\nu_{\text{eff}}}^{+\infty} \frac{S_\omega(\nu)}{\nu^2} d\nu = \frac{1}{\pi} \quad (\text{S.6.1})$$

S.7 Dependence of λ_{ps} with reference frequency detuning

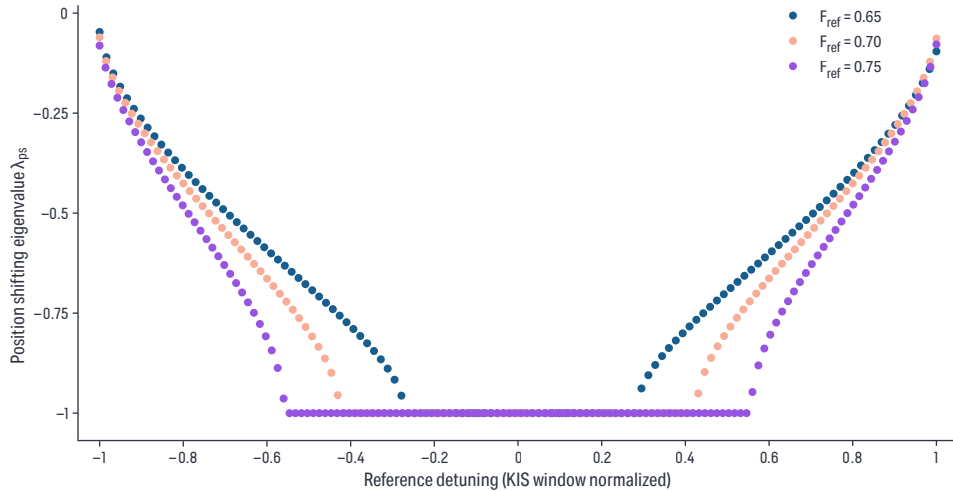


Fig. S.3 – Dependence of λ_{ps} on the reference frequency within the KIS window (normalized to 1) for reference power $F_{\text{ref}} = 0.6, 0.7$ and 0.75 . The position-shifting eigenvalues exhibit a clamping region at -1 , whose width is reference power dependent.

We investigate the value of the position-shifting eigenvalues λ_{ps} as a function of the reference frequency detuning within the KIS window for different reference powers. An important finding is that λ_{ps} is very nonlinear with the reference frequency, leading to a clamping at its minimal value of -1 over a large range of frequencies, a span that is

reference power dependent. This nonlinear nature is particularly important since the residual intensity noise transduces into noise in the KIS window.

Therefore, outside of the -1 clamping, we expect RIN to transduce into λ_{ps} noise, hence noise into the KIS intracavity damping capacity, while it becomes insensitive to RIN within the clamping region.

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