# Quantitative measurement of timing and phase dynamics in a mode-locked laser

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We present results of an experimental study of the timing and phase dynamics in a mode-locked Ti:sapphire laser. By measuring the response of two widely spaced comb lines to a sinusoidal modulation of the pump power, we determine quantitatively the response of both the central pulse time and the phase. Because of the distinct response of the pulse energy, central frequency, and gain to the modulation, we are able to distinguish their contributions to the timing and phase dynamics. © 2007 Optical Society of America

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The ability to control the phase of femtosecond pulses from mode-locked lasers [1] has advanced optical metrology [2] and enabled optical clocks [3]. These applications rely on the laser's narrow comb linewidth, whose magnitude depends mainly on the timing and phase jitter of the pulse train. Recently, there has been interest in how the dynamics of mode-locked lasers affect the properties of the comb [4–6]. Currently, technical noise is the main source of linewidth broadening, but the fundamental limit is set by amplified spontaneous emission [7]. The quantum-noise-limited comb line shape has not been determined. While the statistical properties of the quantum-noise sources are well understood, the linear response of the mode-locked pulse parameters to perturbations must also be known to calculate the line shape. This linear response has not been known, except qualitatively, and in this paper and an earlier paper [8] we directly measure this response. In our earlier work, we showed that the gain must be included among the parameters whose response is considered, and we quantitatively measured the response of the pulse energy, the central frequency, and the round-trip gain to a perturbation. Here, we measure the coefficients that couple those three parameters to the central pulse time and the phase. With these results, we have enough information to fully characterize the response of the laser pulse parameters to perturbations and hence to predict the comb line shape of the laser studied. In addition, this linear response plays a key role in determining the pulse stability. Thus, this study also has important implications for laser design.

Previously, we used a sudden modulation of the pump power and studied the response of the pulse energy and central frequency in the time domain [8]. Here, we use a sinusoidal modulation and examine the response of the laser parameters in the frequency domain. We have verified that such a swept sine technique gives the same results for the energy and central frequency as in [8]. Because the sinusoidal waveform has no low-frequency components, it does not affect the feedback loop central to the technique described here for measurement of timing and phase. When the pump power of the laser is modulated at frequencies near the relaxation oscillation frequency (approximately 400 kHz in the laser studied here), the gain, intensity, and central frequency all have different responses. Thus, we may distinguish their contributions to changes in the central pulse time and phase.

The pulses are characterized by four parameters: the pulse energy $w=w_{\text{eq}}+\Delta w$, the central frequency $\varpi=\varpi_{\text{eq}}+\Delta\varpi$, the central pulse time $\tau=\tau_{\text{eq}}+\Delta\tau$, and the phase $\theta=\theta_{\text{eq}}+\Delta\theta$, where $x_{\text{eq}}$ denotes the equilibrium value of each quantity and $\Delta x$ its change. Because of the system's invariance under time and phase translations, we may choose $\tau_{\text{eq}}=0$. We choose $\theta_{\text{eq}}=2\pi f_0 T$, where $f_0$ is the equilibrium carrier-envelope offset frequency. Following common practice in soliton theory, the phase is defined with respect to a common reference rather than the pulse envelope. With this definition, movement of a mirror in the laser cavity results in changes in both $\tau$ and $\theta$, and the carrier-envelope phase slip $\Delta\theta_{\text{ce}}$ is related to $\Delta\theta$ and $\Delta\tau$ by $\Delta\theta=-\Delta\theta_{\text{ce}}+\varpi\Delta\tau$. To these four pulse parameters we add the round-trip gain $g=g_{\text{eq}}+\Delta g$ as a dynamic variable [8]. The linear response of the laser to perturbations is then [9]

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=-\mathsf{A}\cdot\mathbf{v}+\mathbf{S}, \tag{1}$$

where $\mathbf{v}=(\Delta g,\Delta w,\Delta\varpi,\Delta\tau,\Delta\theta)^t$, $\mathbf{S}$ is a vector of noise sources for each parameter, and $\mathsf{A}$ is a matrix of coefficients that describe the linear response of each parameter to changes in itself or the others. Once $\mathsf{A}$ and $\mathbf{S}$ are known, one can calculate the timing and phase jitter [9], which leads to the optical frequency comb linewidth [10,11]. Here, we measure the elements of $\mathsf{A}$ that couple $w$, $g$, and $\varpi$ to $\tau$ and $\theta$. As $\tau$ and $\theta$ cannot drive $w$, $g$, and $\varpi$ due to time and phase invariance of the system, $A_{x\tau}$ and $A_{x\theta}$ are zero.

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The nonthermal change in the repetition rate $f_{\text{rep}}$ caused by a few percent modulation of the pump power is less than 1 Hz, making an electronic measurement of the pulse timing dynamics over a few $\mu$s extremely difficult. Microstructured fiber used for self-referenced detection [1] of the carrier-envelope offset frequency $f_0$ converts intensity fluctuations into phase (and thus frequency) fluctuations [12], complicating the use of that technique for measurement of the phase dynamics. To circumvent both of these problems, we instead measured the response of comb lines by monitoring the heterodyne beat between the laser being perturbed and a reference laser. The multiplication up to optical frequencies magnifies small changes in $\tau$. By measuring the response of two widely separated comb lines, we can separate the contributions from changes in $\tau$ and $\theta$.

The linear cavity, Kerr lens model-locked Ti:sapphire laser under study had a repetition rate of 93.5 MHz, with dispersion compensation provided by double chirped mirrors and $CaF_2$ prisms. The spectrum was centered at 826 nm with a width of 80 nm. It was pumped with a 532 nm diode-pumped solid state laser that passed through an acousto-optic modulator that could deflect up to 5% of the power with a bandwidth of approximately 5 MHz. The pump power before the acousto-optic modulator was 5.5 W. The modulation frequency was swept from 50 to 800 kHz and the modulation depth was approximately 0.3%. We measured the pump power as a function of time to normalize by the modulation amplitude and phase. We also captured the laser power, spectrally resolved with a monochromator, as a function of time using a photodiode. The integrated spectrum and centroid give the pulse energy and central frequency, respectively.

We locked the tenth harmonic of the repetition rate of the laser under study to the 935 MHz repetition rate of a Ti:sapphire ring laser. The bandwidth of the feedback loop was restricted to 10 kHz to ensure that the loop could not compensate for changes in $f_{\text{rep}}$ from the modulation in the pump power. The actuator for the cavity length was a piezo mounted on the high reflector of the laser under study. Light from both lasers was coupled into single mode fibers and combined to generate an optical heterodyne beat. For two unperturbed lasers with harmonically related repetition rates, the beat frequency is the difference in $f_0$ (modulo the lowest repetition rate) between the two lasers. The carrier-envelope offset frequencies of both lasers were free to drift but did not change significantly over a few $\mu$s, the time scale of the experiment. We captured the heterodyne beat with an oscilloscope and used a nonlinear fitting routine to find the beat frequency as a function of time.

The frequency of the $n$th comb line is $f_n = n f_{\text{rep}} + f_0$. The change in the frequency of the heterodyne beat is equal to the absolute value of the change in the comb line frequency $|\Delta f_n|$. A change in $\tau$ leaves the central comb line near $\varpi_{\text{eq}}$ fixed and moves the $n$th line by a value proportional to $f_n - f_c - f_0$, where $f_c$ is the central frequency, assumed here to be the centroid of the spectrum. A change in $\theta$ affects all comb lines equally. Given changes in two comb lines of frequency $f_n$ and $f_m$, the rate of change of $\tau$ is

$$\Delta \dot{\tau} = \frac{\Delta f_n - \Delta f_m}{f_n - f_m}. \qquad (2)$$

The further apart the comb lines, the more sensitive the measurement of $\tau$. The rate of change of $\theta$ is

$$\Delta \dot{\theta} = 2\pi \frac{\Delta f_n (f_m - f_c - f_0) - \Delta f_m (f_n - f_c - f_0)}{f_n - f_m}. \qquad (3)$$

We used a monochromator with a bandwidth of 1 nm to select two groups of comb lines at 350 and 390 THz. There is an ambiguity in the sign of the change in $f_n$ and $f_m$ because of the beating down to radio frequencies. Results are shown in Fig. 1.

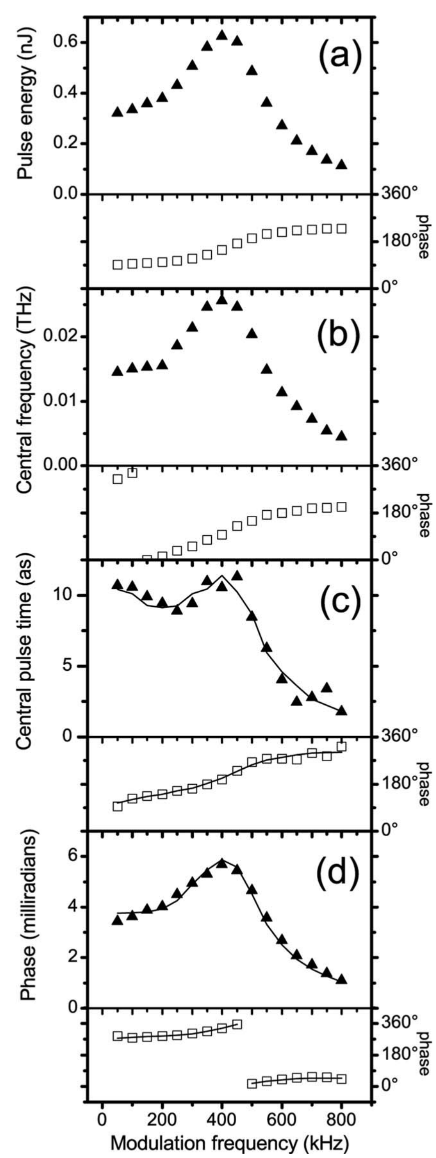


Fig. 1. Experimental results for (a) pulse energy $w$, (b) central frequency $\varpi$, (c) central pulse time $\tau$, and (d) phase $\theta$. The amplitude (triangles) and phase (squares) of the response of each variable are plotted, as well as fits (solid curves) in (c) and (d) to Eq. (4) using the measured response of $w$ and $\varpi$.

Under the assumption that changes in $\theta$ and $\tau$ do not affect the other variables, their response is

$$\frac{\Delta\dot{\tau}(\Omega)}{T_R} = -A_{\tau w}w_0(\Omega) - A_{\tau g}g_0(\Omega) - A_{\tau\varpi}\varpi_0(\Omega),$$

$$\frac{\Delta\dot{\theta}(\Omega)}{T_R} = -A_{\theta w}w_0(\Omega) - A_{\theta g}g_0(\Omega) - A_{\theta\varpi}\varpi_0(\Omega), \quad (4)$$

where $w_0$, $g_0$, and $\varpi_0$ are the complex amplitudes of those variables as functions of the angular modulation frequency $\Omega$. Therefore, one can extract the $A_{xy}$ coefficients by fitting the measured $\tau_0$ and $\theta_0$ to $w_0$, $g_0$, and $\varpi_0$. We used the measured $w_0$ and $\varpi_0$, shown in Figs. 1(a) and 1(b), and used a theoretical model for $g_0$ (the frequency domain version of what was described in [8]).

We assigned the sign of the extracted $A_{xy}$ coefficients using the known sign of the dispersion using $A_{\tau\varpi} = -D/T_R$, where $D = -\beta''$ is the average dispersion in the cavity [9]. We find $A_{\tau\varpi} = -4.0\times10^{10}$ as/(s THz), $A_{\tau w} = 2.0\times10^9$ as/(s nJ), and $A_{\theta w} = 9.4\times10^5$ rad/(s nJ). The data are consistent with the gain's having no direct effect on either $\tau$ or $\theta$ and with the central frequency having no effect on $\theta$, i.e., $A_{\theta\varpi} = A_{\theta g} = A_{\tau g} = 0$. The coefficients coupling pulse energy to $\tau$ and $\theta$ are related to the Kerr nonlinearity in the Ti:sapphire crystal. The Kerr phase shift leads to a nonzero $A_{\theta w}$, while Kerr shock [13,14] contributes to a nonzero $A_{\tau w}$. Other processes may also contribute.

The variation of $f_0$ with pump power is often used to lock it [15,16]. However, many competing processes are involved, some of which affect $\tau$ and others $\theta$ (both play a role in the phase evolution because $\Delta\theta_{ce} = -\Delta\theta + \varpi\Delta\tau$). In addition to the direct changes in phase and timing caused by the Kerr effect (the Kerr phase shift and shock, respectively), gain- and energy-driven frequency pulling changes $\varpi$ [8], which changes $\tau$ through dispersion. The technique described here allows quantitative measurement of these different processes. Using $\mathrm{d}\theta_{ce}/\mathrm{d}w = A_{\theta w} - \varpi A_{\tau w}$, we note that we observe a partial cancellation of the intensity related effects to $\Delta f_0$, as predicted previously [13,14].

In summary, we have developed a comb-based technique for sensitive measurement of the linear response of the timing and phase. By taking advantage of the differing response functions of the pulse energy and central frequency, we were able to fully characterize the timing and phase response of the laser. These results, along with those presented in [8], provide a quantitative measurement of the linear response matrix A. An accurate calculation of the quantum-noise-limited comb line shape for this laser is now possible. Additionally, we expect that the unique ability of the technique described here to distinguish between changes in the timing and phase and identify the sources of these changes will prove useful in the study of other types of mode-locked lasers.

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