Comparison of Two Biasing Monte Carlo Methods for Calculating Outage Probabilities in Systems With Multisection PMD Compensators

Aurenice O. Lima, Curtis R. Menyuk, and Ivan T. Lima, Jr.

***Abstract*—We evaluate the performance of single-section and three-section polarization-mode-dispersion (PMD) compensators using the biasing Monte Carlo methods of importance sampling (IS) and multicanonical Monte Carlo (MMC). We show that standard IS that biases only first-order PMD is insufficient to compute penalties in most compensated systems, while multiple IS that biases both first- and second-order PMD and MMC work well with all the compensators that we investigated. We show that multiple IS works well even in a system with a three-section compensator, when both first- and second-order PMD are compensated. The applicability of IS in these systems is consistent with the existence of a large correlation between first- and second-order PMD of the transmission line and higher orders of PMD after compensation, so that the first two orders, even when compensated, remain highly correlated with the residual penalty. We directly demonstrate the existence of this correlation.**

***Index Terms*—Importance sampling (IS), multicanonical Monte Carlo (MMC) simulations, outage probability, polarization-mode-dispersion (PMD) compensators.**

I. Introduction

POLARIZATION-MODE dispersion (PMD) is a major source of impairment in optical fiber communication systems. Since PMD is a random process, Monte Carlo simulations are often used to compute PMD-induced penalties. However, the large PMD penalties of interest to system designers cannot be efficiently computed using standard, unbiased Monte Carlo simulations, since they are very rare. For example, a designer might require that a penalty larger than 1 dB occurs with probability 10^{-6} or less, which would require on the order of 10^{7} standard Monte Carlo samples or more to simulate. To overcome this hurdle, advanced Monte Carlo methods such as importance sampling (IS) [1], [2] and multicanonical Monte Carlo (MMC) [3] have recently been applied to compute these penalties [4], [5] using a much smaller number of samples.

In optical fiber communication systems without PMD compensators, the penalty is correlated with the differential group delay (DGD) due to PMD. As a consequence, one can apply IS to bias the DGD [1] for the computation of PMD-induced penalties. However, biasing the DGD alone is inadequate to compute penalties in compensated systems. On the other hand, the use of multiple IS in which both first- and second-order PMD are biased [2] allows one to efficiently study important rare events with large first- and second-order PMD. In [4] and [6], we used multiple IS to bias first- and second-order PMD to compute the outage probability due to PMD in uncompensated systems and in compensated systems with a single-section compensator. The development of IS requires some *a priori* knowledge of how to bias a given parameter in the simulations. In this particular problem, the parameter of interest is the penalty. However, to date there is no IS method that directly biases the penalty. Instead of directly biasing the penalty, we relied on the correlation of the first- and second-order PMD with the penalty, which may not hold in all compensated systems. In contrast to IS, MMC does not require *a priori* knowledge of which rare events contribute significantly to the penalty distribution function in the tails. MMC is an iterative method, where in each iteration it produces a biased random walk that automatically searches the state space for the important rare events. This knowledge is accumulated, allowing the penalty distribution function to be obtained further out in the tail from one iteration to the next. In this work, we use multiple IS and MMC to study the performance of single-section and three-section PMD compensators. We show that both methods are appropriate to compute outage probabilities with the compensators that we investigated. They yield the same results within the limit of their statistical errors, and multiple IS yields lower errors for comparable run times.

We investigated a single-section PMD compensator, which is a variable-DGD compensator that was programmed to eliminate the residual DGD at the central frequency of the channel after compensation, and a three-section PMD compensator proposed in [7], which compensates for first- and second-order PMD. We describe details of the implementation of the single-section compensator that we used in a previous publication [6]. The three-section compensator consists of two fixed-DGD elements that compensate for the second-order PMD and one variable-DGD element that eliminates the residual DGD at the central frequency of the channel after compensation. The three-section compensator that we use has the first- and second-order PMD as feedback parameters. This compensator can also, in principle, operate in the feedforward configuration.

II. Simulation Results and Discussions

We evaluate the performance of a single-section and a three-section PMD compensator in a 10-Gb/s nonreturn-to-zero system with a mean DGD of 30 ps. We investigated other values for the mean DGD, spanning the range from 30 to 40 ps and obtained similar results. We use perfectly rectangular pulses

Manuscript received May 23, 2005; revised August 17, 2005. This work was supported in part by the Department of Energy and the National Science Foundation (NSF). The work of I. T. Lima, Jr. was supported by the NSF under Grant 01322899.

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Digital Object Identifier 10.1109/LPT.2005.859183

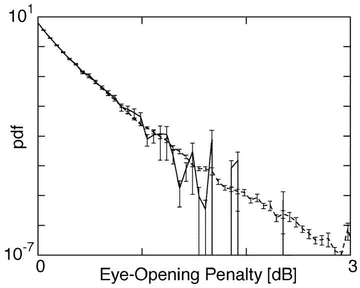

Fig. 1. PDF of the eye-opening penalty for a system with a mean DGD of 30 ps and a single-section compensator. (a) Solid line: results using IS in which only the DGD is biased. (a) Dashed line: results using IS in which both first- and second-order PMD are biased. The confidence interval is shown with error bars.

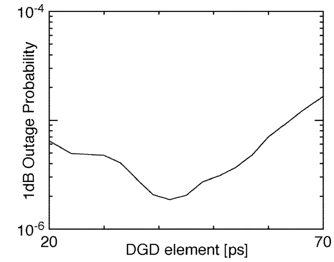

Fig. 2. Outage probability for a 1-dB penalty as function of the DGD element (τ_c) of the three-section compensator for a system with mean DGD of 30 ps.

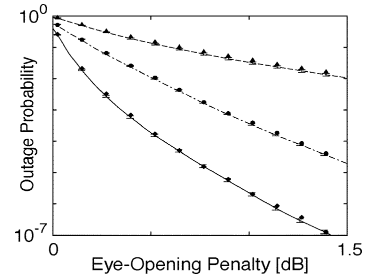

Fig. 3. Outage probability as a function of the eye-opening penalty for a system with mean DGD of 30 ps. (a) Dashed line (MMC) and triangles (IS): uncompensated system. (b) Dotted–dashed line (MMC) and circles (IS): system with a single-section compensator. (c) Solid line (MMC) and diamonds (IS): system with a three-section compensator. The error bars show the confidence interval for the MMC results.

filtered by a Gaussian shape filter that produces a rise time of 30 ps. We use a bit string with an 8-bit de Bruijn sequence. It is sufficient to simulate eight bits, which has all possible three-bit combinations, because we are studying a single-channel system in which the probability of intersymbol interference beyond one bit is negligible. We model the fiber using the coarse step method with 80 birefringent fiber sections, which reproduces first- and higher order PMD distortions within the probability range of interest [6]. The results of our simulations can also be applied to 40-Gb/s systems by scaling down all time quantities by a factor of four. To evaluate the performance, we use the eye opening, which is defined as the difference between the currents in the lowest mark and the highest space at the sampling time [6]. The three-section compensator has two fixed-DGD elements of 45 ps and one variable-DGD element. The results obtained with the three-section compensation for each fiber realization are based on the method described in [7]. In our simulations, we computed the reduction of the polarization chromatic dispersion and the principal states of polarization rotation rate components for the two operating points [7] of the compensator. Then, we selected the one that presented the largest reduction of the second-order PMD. Note that these results are not obtained using a tracking system. The results that we present here were obtained using 30 MMC iterations with 8000 samples each and using IS with a total of 2.4×10^5 samples. We estimate the errors in MMC using a transition matrix method that will be described in detail elsewhere, while we estimate the errors in IS as in [4].

In Fig. 1, we show the probability density function (pdf) of the eye-opening penalty for a system with 30-ps mean DGD and a single-section PMD compensator. We compute the pdf using IS in which only the DGD is biased, and we also compute the pdf using IS in which both the first- and second-order PMD are biased. We observed that it is not sufficient to only bias the DGD in order to accurately calculate the compensated penalty and its pdf. This approach can only be used in systems where the DGD is the dominant source of penalties, which is the case in uncompensated systems and in systems with limited PMD compensation.

In Fig. 2, we plot the outage probability for a 1-dB penalty as function of the DGD element (τ_c) for a system with the three-section compensator that we used. The outage probability is the complement of the cumulative density function (cdfc) of the eye-opening penalty ρ, where $\mathrm{cdfc}(\rho) = \int_\rho^\infty p(\rho')d\rho'$ and $p(\rho)$ is the corresponding pdf. The mean DGD of the system before compensation is 30 ps. We observed that there is an optimum value for τ_c that minimizes the outage probability, which is close to 45 ps. We set the values for the two fixed-DGD elements of the three-section PMD compensator to this optimum value. The reason why the outage probability rises when τ_c becomes larger than this optimum is because large values of τ_c add unacceptable penalties to fiber realizations with relatively small second-order PMD values that could be adequately compensated at lower values of τ_c. We also observed that there is a relatively small dependence of the outage probability on τ_c. That is because the third variable-DGD section of the compensator cancels the residual DGD after the first two sections, which significantly mitigates the penalty regardless of the value of τ_c.

In Fig. 3, we plot the outage probability $(\hat{P}_{\mathrm{op}})$ as a function of the eye-opening penalty for the compensators that we study. The histogram of the penalty was divided into 34 evenly spaced bins in the range -0.1 and 2 dB, even though we show results from 0 to 1.5 dB of penalty. The maximum relative error $(\hat{\sigma}_{\hat{P}_{\mathrm{op}}}/\hat{P}_{\mathrm{op}})$ for the curves computed with MMC shown in this plot equals 0.13. The relative error for the curves computed with IS is smaller than with MMC, and is not shown in the plot. The maximum relative error for the curves computed with IS equals 0.1. The results obtained using MMC (solid lines) are in agreement with the ones obtained using IS (symbols). The agreement between the MMC and IS results was expected for the case that we use a single-section compensator, since this type of compensator can only compensate for first-order PMD [5], so that the dominant source of penalty after compensation is the second-order PMD of the transmission line. Hence, it is expected that MMC and IS give similar results. We also observe good agreement between the MMC and IS results for the three-section compensator. This level of agreement is an indication that three-section compensators that compensate for the first two orders of the Taylor expansion of the transmission line PMD produce residual third and higher orders of PMD that are significantly correlated with the first- and second-order PMD

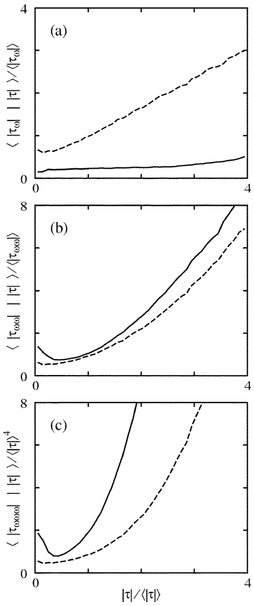

Fig. 4. (a) Conditional expectation of the magnitude of the normalized second-order PMD, $|\boldsymbol{\tau}_\omega|$, given a value of the DGD of the transmission line, $|\boldsymbol{\tau}|$. Conditional expectation before (dashed) and after (solid) the three-section compensator. (b) Same as (a) for the normalized third-order PMD $|\boldsymbol{\tau}_{\omega\omega}|$. (c) Same as (a) for the normalized fourth-order PMD $|\boldsymbol{\tau}_{\omega\omega\omega}|$.

of the transmission line. That is why the use of IS to bias first- and second-order PMD is sufficient to accurately compute the outage probability in systems where the first two orders of PMD of the transmission line are compensated.

In Fig. 4, we quantify the correlation between lower and higher orders of PMD. In Fig. 4(a), we show the conditional expectation of the magnitude of second order of PMD before and after the three-section compensator given a value of the DGD of the transmission line. We normalize the DGD $|\boldsymbol{\tau}|$ by the mean DGD $\langle|\boldsymbol{\tau}|\rangle$ and $|\boldsymbol{\tau}_\omega|$ by $\langle|\boldsymbol{\tau}_\omega|\rangle$ to obtain results that are independent of the mean DGD and of the mean of the magnitude of second-order PMD. We observe a large correlation between $|\boldsymbol{\tau}|$ and $|\boldsymbol{\tau}_\omega|$ before compensation, while after compensation $|\boldsymbol{\tau}_\omega|$ is significantly reduced and is less correlated with the DGD, demonstrating the effectiveness of the three-section compensator in compensating for second-order PMD. In Fig. 4(b) and (c), we show the conditional expectation of the magnitude of the third-order PMD and of the fourth-order PMD, respectively, before and after the three-section compensator, given a value of the DGD of the transmission line. In both cases, we observed a high correlation of the third- and the fourth-order PMD with the DGD before and after compensation. In addition, we observed a significant increase of these higher order PMD components after compensation, which leads to a residual penalty after compensation that is correlated to the original first- and second-order PMD. The second-order PMD is also correlated to both third- and fourth-order PMD before and after compensation. These correlations are not shown in the manuscript due to limited space.

In Fig. 5, we show contour plots of the conditional expectation of the penalty with respect to the first- and second-order PMD for a system with a three-section PMD compensator [7]. These results show that the residual penalty after compensation is significantly correlated with the first- and second-order PMD. The correlation between the higher orders of PMD with the DGD that we show in Fig. 4(a)–(c) can be estimated from the

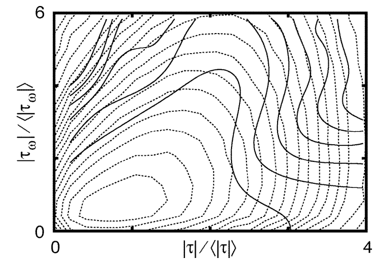

Fig. 5. Three-section compensated system. The dotted lines are the contour plots of the joint pdf of the normalized $|\boldsymbol{\tau}|$ and $|\boldsymbol{\tau}_\omega|$ (from bottom to top) at 3×10^{-n}, with $n = 1, 2, 3, 4, 5, 6, 7$, and 10^{-m}, with $m = 1, \ldots, 11$. The solid lines are the contour plots of the conditional expectation of the eye-opening penalty in decibels (from bottom to top) at 0.1, 0.2, 0.3, 0.4, 0.5, 0.6.

concatenation rule [8], which explicitly indicates a dependence of the higher order PMD components on the lower order components. The increase in these higher order components after compensation is also due to our choice of operating point for this compensator, which is set to compensate only for first- and second-order PMD, regardless of the higher order PMD components. It is possible that this three-section PMD compensator would perform better if all seven parameters of the compensator are adjusted to achieve the global optimum of the penalty reduction. However, finding this global optimum is unpractical due to the large number of local optima in such a multidimensional optimization space, as we found in our investigation of single-section PMD compensators [6].

III. Conclusion

We have shown that both multiple IS and MMC can be used to bias Monte Carlo simulations of the outage probability due to PMD in optical fiber communication systems with both one-section and three-section compensators. In particular, multiple IS can be used to efficiently compute the outage probability even with a three-section PMD compensator in which both first- and second-order PMD are compensated, which is consistent with the presence of a large correlation between first- and second-order PMD of the transmission line and higher orders of PMD after compensation. We directly demonstrated that this correlation is present. Finally, we showed that MMC yields the same results, within the statistical errors of both methods.

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