

# Comparison of penalties resulting from first-order and all-order polarization mode dispersion distortions in optical fiber transmission systems

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We compare the eye-opening penalty from a first-order polarization mode dispersion (PMD) model with that from an all-order PMD model in optical fiber transmission systems. Evaluating the performance by taking into account only first-order PMD produces a good approximation of the true eye-opening penalty of uncompensated systems when the penalty is low. However, when the penalties are high, this model overestimates the penalty for outage probabilities in the range of interest for systems designers, which is typically approximately  $10^{-5}$  to  $10^{-6}$ . © 2003 Optical Society of America

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Polarization mode dispersion (PMD) is one of the barriers to achieving single-channel data rates at 10 Gbits/s and beyond in a significant number of terrestrial optical fiber systems. PMD is characterized by polarization dispersion vector  $\mathbf{\Omega}(\omega)$  in Stokes space. The frequency-dependent evolution of the states of polarization in optical fibers can be described by the vector equation<sup>1</sup>  $\mathbf{s}_\omega = \mathbf{\Omega} \times \mathbf{s}$ , where the subscript  $\omega$  represents the derivative with respect to the angular frequency,  $\omega$ , and  $\mathbf{s}$  is the three-dimensional Stokes vector.

As long as the signal bandwidth is sufficiently narrow, the first-order PMD distortion is the dominant PMD effect. First-order PMD distortion is the distortion that occurs if it is possible to neglect the variation of  $\mathbf{\Omega}(\omega)$  as a function of frequency over the signal bandwidth, replacing  $\mathbf{\Omega}(\omega)$  with  $\mathbf{\Omega}(\omega_c)$ , where  $\omega_c$  is the central frequency of the channel. In this approximation, the signal splits into principal states of polarizations (PSPs),  $\pm\mathbf{\Omega}/|\mathbf{\Omega}|$ , that propagate at different group velocities, causing intersymbol interference. The differential group delay (DGD) between the PSPs is equal to the length of the polarization dispersion vector  $|\mathbf{\Omega}|$ .

In the first-order approximation, the PSPs and the DGD do not vary with frequency. The correlation bandwidth<sup>1</sup>  $\Delta\nu_{\text{PMD}}$  over which the DGD can be considered constant is inversely proportional to the expected value of the DGD, with a constant of proportionality close to 1/2. For pulses with a small bandwidth compared with  $\Delta\nu_{\text{PMD}}$ , which means that the expected DGD is small compared with the duration of the optical pulse, the first-order model is practically useful.

Because of the simplicity of the first-order PMD model and its practical usefulness, it has frequently been used for performance evaluation of systems.<sup>2-4</sup> In this Letter we show that performance evaluation taking into account only first-order PMD produces a good approximation of the true eye-opening penalty of uncompensated optical systems when the penalty is low. However, such evaluation overestimates the true eye-opening penalty, i.e., the penalty

obtained from a PMD model that takes into account the frequency variation of the DGD and of the PSPs, at the higher penalties that are associated with outage probabilities of  $10^{-5}$  and  $10^{-6}$ .

We now describe the PMD simulation model that we use.<sup>5</sup> This model is based on the representation of the fiber link by a frequency-dependent transfer function matrix,  $\mathbf{T}(\omega)$ , and it takes into account all orders of PMD. We do not take into account polarization-independent loss, chromatic dispersion, or fiber nonlinearity. The electrical field vector at the end of the fiber link,  $\mathbf{E}_{\text{out}}(\omega)$ , equals

$$\mathbf{E}_{\text{out}}(\omega) = \mathbf{T}(\omega) \cdot \mathbf{E}_{\text{in}}(\omega), \quad (1)$$

where  $\mathbf{E}_{\text{in}}(\omega)$  is the input electrical field vector in the Jones space. Using the coarse step model of a fiber,<sup>5</sup> we may write the transfer function,  $\mathbf{T}(\omega)$ , of an optical fiber that consists of  $N$  birefringent sections as<sup>5</sup>

$$\mathbf{T}(\omega) = \prod_{i=1}^N \mathbf{S}_\omega \cdot \mathbf{S}_i, \quad (2)$$

where

$$\mathbf{S}_\omega = \begin{bmatrix} \exp(j\omega|\mathbf{\Omega}|_s/2) & 0 \\ 0 & \exp(-j\omega|\mathbf{\Omega}|_s/2) \end{bmatrix}, \quad (3)$$

$\mathbf{S}_i$

$$= \begin{bmatrix} \cos(\xi_i/2) & j \sin(\xi_i/2) \\ \times \exp[(\psi_i + \phi_i)j/2] & \times \exp[(\psi_i - \phi_i)j/2] \\ j \sin(\xi_i/2) & \cos(\xi_i/2) \\ \times \exp[-(\psi_i - \phi_i)j/2] & \times \exp[-\psi_i + \phi_i)j/2] \end{bmatrix}, \quad (4)$$

where  $|\mathbf{\Omega}|_s$  is the DGD in a single section and  $\xi_i$ ,  $\psi_i$ , and  $\phi_i$  are random variables that are independent at

each  $i$  and from one another. The probability density functions of angles  $\psi_i$  and  $\phi_i$  are uniformly distributed from 0 to  $2\pi$ , and those of the  $\cos \xi_i$  are uniformly distributed from  $-1$  to  $1$ . We set  $|\Omega|_s$  as<sup>6</sup>

$$|\Omega|_s = \sqrt{3\pi/8N} \langle |\Omega| \rangle, \quad (5)$$

where  $\langle |\Omega| \rangle$  is the expected DGD. In the first-order PMD model, we set  $N = 1$ , and in the all-order model we set  $N = 80$ . Previous work<sup>7</sup> indicated that  $N = 80$  is sufficient to yield a Maxwellian distribution of the DGD in the outage probability range  $10^{-5}$ – $10^{-6}$ . We assume that the fiber passes ergodically through all possible orientations of the birefringence.

Figures 1–3 show the outage probability of the eye-opening penalty caused by PMD in an uncompensated 10-Gbit/s nonreturn-to-zero (NRZ) system. We model the receiver as an ideal square-law photodetector followed by a fifth-order electrical Bessel filter.

We evaluate the effects that PMD has on the system performance by use of eye opening, which we define as the difference between the currents in the lowest mark and the highest space at the decision time. The eye-opening penalty is the ratio between the back-to-back and the PMD-distorted eye opening, expressed in decibels. In optical fiber systems, designers specify an eye-opening penalty margin for PMD (for example, 1 dB), and they want to ensure that the eye-opening penalty that is due to PMD will exceed this margin with a very low probability. This probability is referred to as the outage probability. We use importance sampling applied to PMD to study events with low probability efficiently.<sup>7,8</sup>

In Fig. 1, we compare the eye-opening penalty from a first-order PMD model with that of an all-order PMD model when the mean DGD  $\langle |\Omega| \rangle$  is equal to 14 ps, for a typical NRZ system whose pulses have rise and fall times of 30 ps and whose receiver has an electrical filter with a FWHM bandwidth equal to 8.6 GHz. For this low value of  $\langle |\Omega| \rangle$  the system penalty is dominated by first-order PMD, so the eye-opening penalty when we consider all orders of PMD is very close to the eye-opening penalty when only the first order is considered. The PMD-induced penalty is thus highly correlated with the DGD at the central frequency of the channel.<sup>7</sup> In this case, the performance evaluation when we take into account only first-order PMD produces a good approximation of the true eye-opening penalty.

In Fig. 2, we show the curves of outage probability versus eye-opening penalty for the same system as in Fig. 1 with  $\langle |\Omega| \rangle = 20$  ps. In Fig. 2, we observe that the first-order PMD model yields an eye-opening penalty that is slightly greater than that of the all-order PMD model, for outage probability of the order of  $10^{-6}$ .

Finally, in Fig. 3, we show the eye-opening penalty results when we compare first- and all-order PMD in a NRZ system whose pulses have rise and fall times of 5 ps and whose receiver has an electrical filter with a FWHM bandwidth equal to 10 GHz. As we can see in Fig. 3, the difference in eye-opening penalty values between the first- and all-order models

is larger than in the previous case illustrated in Fig. 2 because the pulse format used in this system has a broader bandwidth. In Figs. 2 and 3, we observe that as higher-order PMD effects become more important, the true eye-opening penalty tends to be smaller than the penalty produced by the first-order PMD model. In particular, in Fig. 3, the first-order PMD model overestimates the true eye-opening penalty that is due

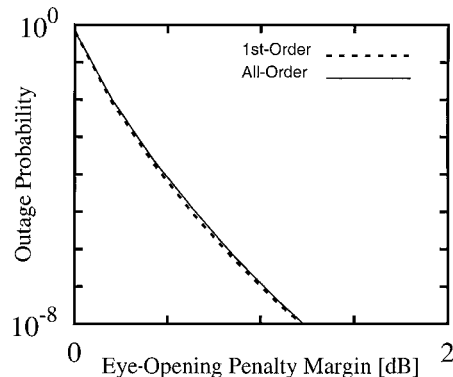


Fig. 1. Outage probability as a function of the eye-opening penalty margin for a 10-Gbit/s NRZ system with pulse edge rise and fall times of 30 ps and mean DGD,  $\langle |\Omega| \rangle$ , equal to 14 ps. The dashed curve is the outage probability when only first-order PMD is considered, and the solid curve is the result when all-order (first- and higher-order) PMD distortion is considered. The outage probability is the probability that the eye-opening penalty exceeds the value displayed on the horizontal axis.

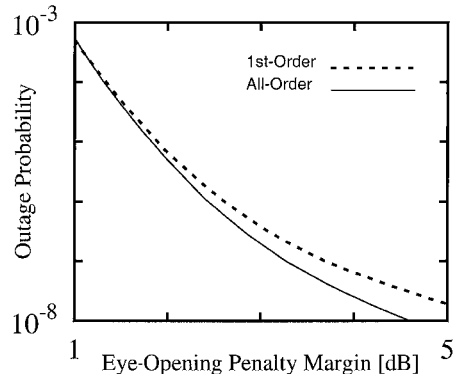


Fig. 2. Same as Fig. 1, except that  $\langle |\Omega| \rangle = 20$  ps and the pulse edge rise and fall times are 30 ps.

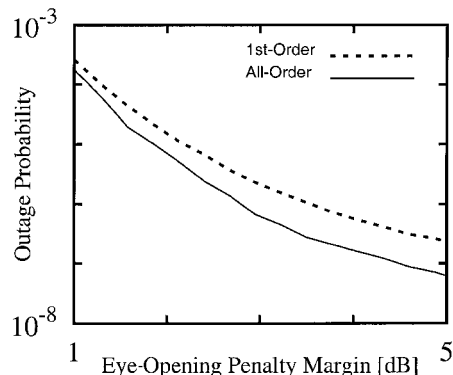


Fig. 3. Same as Fig. 1, except that  $\langle |\Omega| \rangle = 20$  ps and the pulse edge rise and fall times are 5 ps.

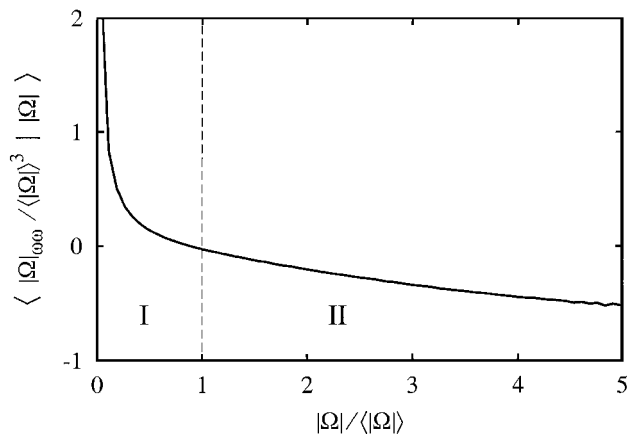


Fig. 4. Conditional expectation of the second derivative of the DGD,  $|\Omega|_{\omega\omega}$ , given a value of the DGD,  $|\Omega|$ .

to all orders of PMD for the outage probability value of  $10^{-6}$  by approximately 0.8 dB.

To understand why the first-order PMD model overestimates the penalty for outage probabilities in the range of interest for system designers, which is typically approximately  $10^{-5}$  to  $10^{-6}$ , we study how the DGD varies as a function of the angular frequency,  $\omega$ , in the neighborhood of a frequency  $\omega_c$  where the DGD is large. In general, the DGD has a nonzero slope that is due to first- and second-order PMD. However, when the DGD is large, the sign of the slope has no effect on the DGD averaged over the bandwidth of the signal when the pulse's frequency spectrum is symmetric. Consequently, the slope of the DGD will have a small effect on the penalty. Moreover, since the effect of the slope causes a small increase in the penalty as often as a small decrease, the expected penalty will depend only weakly on the slope when the DGD is large. In general, the DGD will also have a curvature that is due to the first three orders of PMD. For large DGD, this curvature will be negative, reducing the DGD when it is averaged over the bandwidth of the signal, more often than it will be positive. Thus, the expected penalty will be reduced. To quantify this observation, we express the curvature of the DGD,  $|\Omega|_{\omega\omega}$ , which depends on the first three orders of PMD, as

$$|\Omega|_{\omega\omega} = \frac{|\Omega_\omega|^2}{|\Omega|} - \frac{(\Omega \cdot \Omega_\omega)^2}{|\Omega|^3} + \frac{\Omega \cdot \Omega_{\omega\omega}}{|\Omega|}. \quad (6)$$

Using Eq. (6), we calculate the conditional expectation of  $|\Omega|_{\omega\omega}$  at the channel's central frequency,  $\omega_c$ , as a function of the DGD, shown in Fig. 4. We use a total of  $10^6$  Monte Carlo realizations with importance sampling<sup>8</sup> and 80 equally spaced bins in the range  $0 \leq |\Omega| \leq 5$  to generate the curve in Fig. 4. We normalize the DGD  $|\Omega|$  by the mean DGD  $\langle |\Omega| \rangle$  and  $|\Omega|_{\omega\omega}$  by  $\langle |\Omega| \rangle^3$  to obtain results that are independent of the mean DGD of the fiber. This conditional expectation  $\langle |\Omega|_{\omega\omega} | |\Omega| \rangle$  gives the value of  $|\Omega|_{\omega\omega}$  when it is averaged over all the fiber realizations for a given value of  $|\Omega|$ . When the DGD is larger than the

mean, corresponding to region II in Fig. 4, the value of  $\langle |\Omega|_{\omega\omega} | |\Omega| \rangle$  is negative, leading to a reduction of the average DGD when it is averaged both over the bandwidth of the signal and over fiber realizations, which in turn leads to a reduction of the eye-opening penalty margin at a given outage probability.

This explanation, which takes into account only up to the third order of PMD, is merely a heuristic explanation of the results shown in Figs. 1–3, which include effects at all orders of PMD. When the DGD is large, it is reasonable to suppose that orders even higher than third might play a significant role. It is possible that the whole notion of a higher-order PMD expansion, which implicitly assumes that the polarization dispersion vector can be written as a convergent Taylor series over the bandwidth of the signal, may break down at the large DGD values that typically produce unacceptable penalties. The resolution of this issue lies beyond the scope of this Letter. Our results do, however, show conclusively that simple theoretical models that take into account only first-order PMD will overestimate the penalty when the DGD is large. Our work also indicates that third and possibly higher orders of PMD play a significant role in reducing the expected penalty.

In conclusion, we have compared the eye-opening penalty margin from a first-order PMD model with the penalty from an all-order PMD model in optical fiber transmission systems. We showed that performance evaluation taking into account only first-order PMD produces a good approximation of the true eye-opening penalty of uncompensated systems when the penalty is low. However, when the penalties are high, the first-order PMD model actually overestimates the penalty. Thus, the first-order PMD model is conservative compared with the penalty that is obtained by use of a model with first- and higher-order PMD distortions.

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## References

1. C. D. Poole and J. Nagel, in *Optical Fiber Telecommunications III-A*, I. P. Kaminow and T. L. Koch, eds. (Academic, San Diego, Calif., 1997).
2. C. D. Poole, R. W. Tkash, A. R. Chraplyvy, and D. A. Fishman, *IEEE Photon. Technol. Lett.* **3**, 68 (1991).
3. H. Bülow, *Electron. Lett.* **31**, 214 (1995).
4. A. Djupsjöbacka, *IEEE Photon. Technol. Lett.* **13**, 660 (2001).
5. D. Marcuse, C. R. Menyuk, and P. K. A. Wai, *J. Lightwave Technol.* **15**, 1735 (1997).
6. C. D. Poole and D. L. Favin, *J. Lightwave Technol.* **12**, 917 (1994).
7. I. T. Lima, Jr., G. Biondini, B. Marks, W. L. Kath, and C. R. Menyuk, *IEEE Photon. Technol. Lett.* **14**, 627 (2002).
8. G. Biondini, W. L. Kath, and C. R. Menyuk, *IEEE Photon. Technol. Lett.* **14**, 310 (2002).