

# Autocorrelation function for polarization mode dispersion emulators with rotators

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Received February 19, 2002

We derive a recursion relation for the frequency autocorrelation function of the polarization dispersion vector for polarization mode dispersion emulators with rotators. The autocorrelation function has a nonzero background for an emulator with a fixed number of sections. This background diminishes slowly as the number of sections grows. Randomizing the section lengths removes the autocorrelation periodicity exhibited by an emulator with equal sections, but it does not remove the finite background.

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OCIS codes: 260.5430, 060.2330.

Polarization mode dispersion (PMD) is an increasingly important limitation in long-haul optical fiber communications as the per-channel data rate increases, and, consequently, PMD emulators are becoming increasingly important in system design. Emulators are necessary, in part, because high-PMD fiber, which still plays a major role in existing systems, is no longer commercially available. However, even when it is possible to obtain high-PMD fiber, emulators play a useful role, since it is possible to examine a large ensemble of system states far more rapidly than in a test bed with high-PMD fiber.

It was previously suggested that proper emulator design requires two characteristics: (1) Maxwellian distribution of the differential group delay (DGD) and (2) quadratic decay of the autocorrelation function of the polarization dispersion vector.<sup>1,2</sup> Several groups have considered the DGD distribution of PMD emulators,<sup>3–5</sup> but to date there are fewer results for the autocorrelation function. The autocorrelation function gives the bandwidth over which a first-order PMD model is applicable. Hence, accurate emulation of the frequency dependence of PMD in optical fibers is important for realistic wavelength-division multiplexing experiments.

PMD emulators are composed of a number of sections of high-birefringence fiber or wave plates connected either by polarization rotators, which change only the angle between the birefringent axes of successive sections, or by polarization controllers, which perform a completely random scattering on the Poincaré sphere. With only one free parameter,

rotators are more commonly used in emulators than polarization controllers. We present here a concise analytical expression for the autocorrelation function of a general  $N$ -section emulator with rotators. This expression yields results that are consistent with earlier numerical results.<sup>3</sup> In particular, we find that the autocorrelation function has a finite background that diminishes slowly with the number of sections. We then use this formula to demonstrate that it is possible to eliminate frequency-domain periodicity in the autocorrelation function by varying the lengths of the sections. This periodicity is not acceptable if emulators are to be used in wavelength-division multiplexing system studies.

A convenient way to describe polarization in a given system is the polarization dispersion vector,  $\mathbf{\Omega}(\omega)$ , whose direction is parallel to the principal states in Stokes space and whose length is the DGD in the system at frequency  $\omega$ .<sup>6</sup> The autocorrelation function is then given by  $\langle \mathbf{\Omega}(\omega) \cdot \mathbf{\Omega}(\omega_0) \rangle$ , where the angle brackets  $\langle \cdot \rangle$  indicate an ensemble average of realizations.

A general recursion relation for the polarization dispersion vector is given by<sup>7,8</sup>

$$\mathbf{\Omega}^{(n)}(\omega) = \tau_n \hat{e}_n + \mathbf{M}_n(\omega) \mathbf{\Omega}^{(n-1)}(\omega), \quad (1)$$

where  $\mathbf{\Omega}^{(n)}$  is the polarization dispersion vector after  $n$  sections of an emulator,  $\tau_n$  is the DGD in the  $n$ th section,  $\hat{e}_n$  is a unit vector pointing in the direction of the polarization dispersion vector for the  $n$ th section, and  $\mathbf{M}_n$  is the Müller matrix for the  $n$ th section. In the

case of emulators composed of polarization rotators, the Müller matrix for one birefringent section is given by  $M_n(\omega) = R_x[\gamma_n(\omega)]R_z(\theta_n)$ , where  $R_x(\psi)$  and  $R_z(\psi)$  are rotations about the  $x$  and  $z$  axes, respectively, by angle  $\psi$ . In this formulation,  $\gamma_n(\omega)$  is the birefringent phase retardation, which is frequency dependent, and  $\theta_n$  is the angle between the birefringent axes of the  $(n - 1)$ th section and the  $n$ th section, which is assumed to be frequency independent. In this case, the recursion relation simplifies to

$$\Omega^{(n)}(\omega) = \tau_n \hat{e}_1 + R_x[\gamma_n(\omega)]R_z(\theta_n)\Omega^{(n-1)}(\omega), \quad (2)$$

where  $\hat{e}_1$  is the unit vector along the  $+x$  axis. One can formulate the autocorrelation function from this expression by averaging over the angles  $\theta_n$ , which are assumed to be independent and identically distributed random variables uniformly distributed between 0 and  $2\pi$ , yielding

$$f_n(\omega, \omega_0) \equiv \langle \Omega^{(n)}(\omega) \cdot \Omega^{(n)}(\omega_0) \rangle_\theta \\ = \tau_n^2 + \langle \Omega^{(n-1)}(\omega)^T \Lambda_n \Omega^{(n-1)}(\omega_0) \rangle_\theta, \quad (3)$$

where  $\langle \cdot \rangle_\theta$  indicates the average over  $\theta_n$  and where

$$\Lambda_n = \frac{1}{2} (1 + \cos \Delta\gamma_n) \mathbf{I} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} (\cos \Delta\gamma_n - 1) \end{bmatrix}, \quad (4)$$

$\Delta\gamma_n = \gamma_n(\omega) - \gamma_n(\omega_0)$ , and  $\mathbf{I}$  is the  $3 \times 3$  identity matrix. We take  $\gamma_n = \tau_n \omega$  so that  $\Delta\gamma_n = \tau_n(\omega - \omega_0) = \tau_n \Delta\omega$ . Then, the autocorrelation function after  $n$  sections of the emulator is given by

$$f_n(\omega, \omega_0) = \tau_n^2 + A_n f_{n-1}(\omega, \omega_0) + B_n g_{n-1}(\omega, \omega_0), \quad (5)$$

where  $A_n \equiv (\cos \tau_n \Delta\omega + 1)/2$ ,  $B_n \equiv (\cos \tau_n \Delta\omega - 1)/2$ ,  $g_n(\omega, \omega_0) \equiv \langle \Omega_z^{(n)}(\omega) \Omega_z^{(n)}(\omega_0) \rangle$ , and  $\Omega_z$  is the  $z$  component of the  $\Omega$  vector. Using Eq. (2), we also find that

$$g_n(\omega, \omega_0) = C_n f_{n-1}(\omega, \omega_0) + D_n g_{n-1}(\omega, \omega_0), \quad (6)$$

where  $C_n \equiv (\cos \tau_n \Delta\omega - \cos \tau_n \bar{\omega})/4$ ,  $D_n \equiv (\cos \tau_n \Delta\omega + 3 \cos \tau_n \bar{\omega})/4$ , and  $\bar{\omega} = \omega + \omega_0$ . In Eqs. (5) and (6), the initial conditions are  $f_0(\omega, \omega_0) = g_0(\omega, \omega_0) = 0$ .

Equations (5) and (6) give a recursion relation for the autocorrelation function of an emulator with rotators with any set of fixed DGD segments. If all the segments' DGDs,  $\tau_n$ , are the same, then this autocorrelation function exhibits a periodicity that can be eliminated by randomization of the DGDs. To study the effect of the randomization of the segments' DGDs, we set  $\tau_n = \sqrt{3\pi/8N} \langle \text{DGD} \rangle (1 + \sigma x_n)$ , where  $\langle \text{DGD} \rangle$  is the mean DGD of the emulator and  $x_n$  is a random number selected from a Gaussian distribution of zero mean and unit variance. Increasing  $\sigma$  therefore corresponds to adding more randomization of the sections' DGD values. We can use the recursion relations, Eqs. (5) and

(6), to obtain recursion relations for the mean and standard deviation—over  $\tau_n$ —of the autocorrelation function given a value of  $\sigma$ , giving, for the mean,

$$\langle f_n \rangle_\tau = \langle \tau_n^2 \rangle_\tau + \langle A_n \rangle_\tau \langle f_{n-1} \rangle_\tau + \langle B_n \rangle_\tau \langle g_{n-1} \rangle_\tau, \quad (7)$$

$$\langle g_n \rangle_\tau = \langle C_n \rangle_\tau + \langle f_{n-1} \rangle_\tau + \langle D_n \rangle_\tau \langle g_{n-1} \rangle_\tau, \quad (8)$$

where  $\langle \cdot \rangle_\tau$  indicates an average over the Gaussian-distributed random variable,  $x_n$ . Since  $x_n$  is assumed to be independent and identically distributed, it is reasonable to separate the average at the  $n$  level from the average at the  $n - 1$  level in Eqs. (7) and (8). In these expressions,

$$\langle A_n \rangle_\tau = \frac{1}{2} [h(\Delta\omega) + 1], \quad (9)$$

$$\langle B_n \rangle_\tau = \frac{1}{2} [h(\Delta\omega) - 1], \quad (10)$$

$$\langle C_n \rangle_\tau = \frac{1}{4} [h(\Delta\omega) - h(\bar{\omega})], \quad (11)$$

$$\langle D_n \rangle_\tau = \frac{1}{4} [h(\Delta\omega) + 3h(\bar{\omega})], \quad (12)$$

with  $h(\xi) = \cos(\sqrt{3\pi/8N} \langle \text{DGD} \rangle \xi) \exp(-3\sigma^2 \pi \times \langle \text{DGD} \rangle^2 \xi^2 / N)$  and  $\langle \tau_n^2 \rangle_\tau = (3\pi/8N) \langle \text{DGD} \rangle^2 (1 + \sigma^2)$ . The expression for the standard deviation is a bit more complicated, but it can be obtained after calculation of  $\langle f_n^2 \rangle_\tau$ , which is done with a procedure analogous to our calculation of  $\langle f_n \rangle_\tau$ .

In Figs. 1–3, we present the mean autocorrelation function as well as the mean with one standard deviation added and subtracted (dashed curves), for an

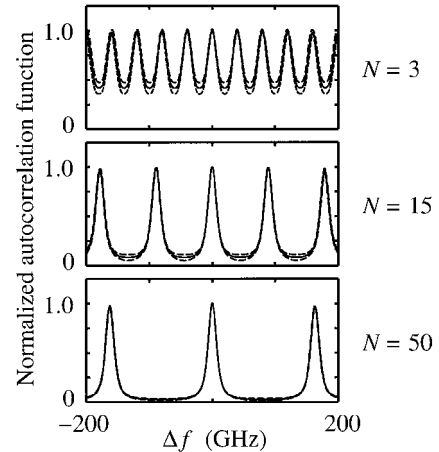


Fig. 1. Mean (solid curves) and mean plus and minus one standard deviation (dashed curves) of the autocorrelation function of emulators with rotators and an average DGD of 40 ps with different numbers  $N$  of sections. The sections' DGDs are Gaussian distributed, with a variance of  $\sigma = 1\%$  of the mean DGD value. The center frequency,  $\Delta f = 0$  GHz, corresponds to the carrier wavelength,  $\lambda_0 = 1.55 \mu\text{m}$ . Each autocorrelation function is normalized by the mean autocorrelation function at  $\Delta f = 0$  GHz.

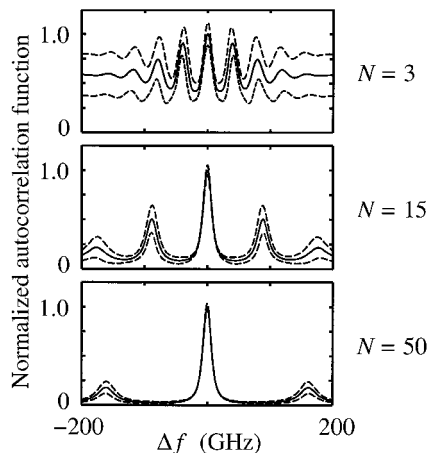


Fig. 2. Same as Fig. 1, except that  $\sigma = 10\%$ .

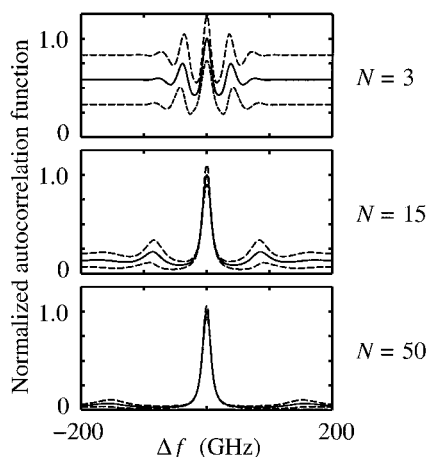


Fig. 3. Same as Fig. 1, except that  $\sigma = 20\%$ .

emulator with  $\langle \text{DGD} \rangle = 40$  ps and with  $N = 3, 15, 50$ . We set  $\sigma = 0.01$  (Fig. 1),  $\sigma = 0.1$  (Fig. 2), and  $\sigma = 0.2$  (Fig. 3). We note that the contributions depending on  $\bar{\omega}$  are nearly negligible in Eqs. (11) and (12). Neglecting them, we find that the frequency difference between the peaks of the autocorrelation

function is proportional to  $1/\langle \text{DGD} \rangle$ . The criterion that we wish to impose on the autocorrelation function is that the background level be as small as possible over a large bandwidth. We have found that the local minimum next to the central peak of the mean autocorrelation function decays like  $1/N$ . The secondary peak adjacent to the central peak also decays like  $1/N$  for large  $N$  and for large  $\sigma$ . Clearly, for the nearly periodic case ( $\sigma = 0.01$ ), the secondary peaks persist for large  $N$ , although the peaks spread in  $\Delta f$  like  $\sqrt{N}$ .

In conclusion, we have provided a closed-form expression for the autocorrelation function of a PMD emulator composed of high-birefringence fiber segments and polarization rotators. We use this expression to demonstrate the effect of randomization of the segments' DGDs. We show that with sufficient randomization, the background level of the autocorrelation function can be reduced, giving a PMD emulator that can be effectively used in WDM experiments.

The authors acknowledge support from the U.S. Department of Energy and the National Science Foundation. B. S. Marks's e-mail address is marks@umbc.edu.

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