Theoretical Investigation of Optical Fiber-Length-Dependent Phase Noise in Opto-Electronic Oscillators

The effects of optical propagation on RF signal and noise

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The Opto-electronic Oscillator

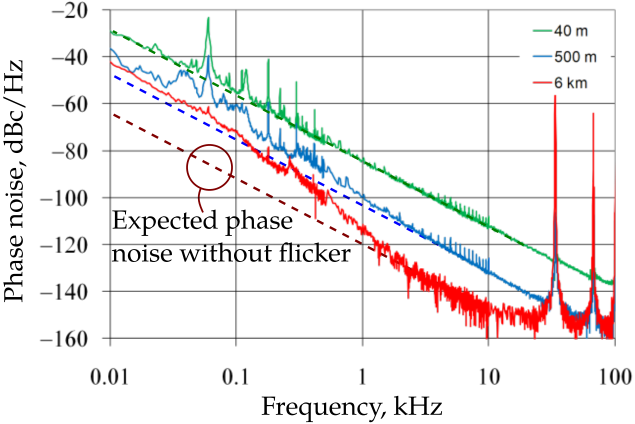
- Opto-electronic oscillators (OEO) operate with low phase noise due to the large delay and low loss of optical fibers.[1]
- OEOs have noise sources in both electronic and optical domains
- Impact on RF photonic devices of noise in optical domain is not well understood
- Length-dependent noise sources dominate for $L > 6$ km to prevent further improvement of phase noise.

What happens to noise in the optical domain?

[1] X. S. Yao and L. Maleki, JOSA B, **8** 1725–35 (1996).

Experimental evidence

- Length-dependent flicker noise is seen experimentally, where does it come from?



OEO: Noise sources

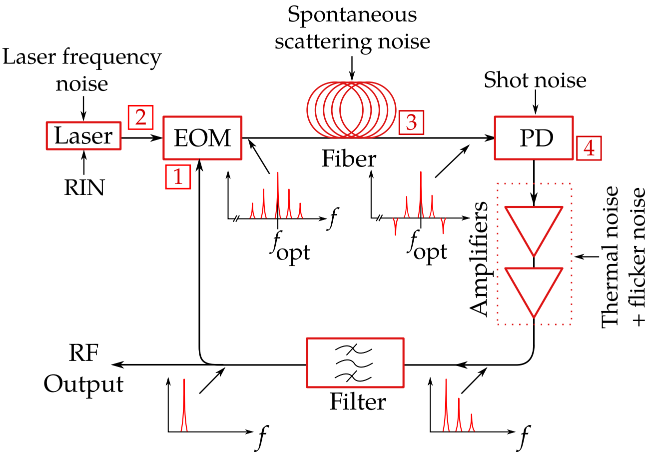


Figure: The OEO system showing the sources of noise and the harmonics of the RF signal at different points in the loop.

The optical path: Modulation

1. The modulator

$$E_{\rm in}(t) = \frac{1}{2}E_{\rm laser}(t)\Big\{\eta_1 \exp[jv_1 A_{\rm in}(t)] + \eta_2 \exp[jv_2 A_{\rm in}(t) + j\psi]\Big\}$$
$$= E_{\rm laser}(t) \sum_{m=-\infty}^{\infty} a_m(t) \exp(jm\omega_0 t)$$

Where the applied RF signal with frequency ω_0 is given by:

$$A_{\rm in}(t) = V_{\rm in} \cos\Big[\omega_0 t + \phi(t)\Big]$$

$V_{\rm in}$: RF amplitude
ϕ: RF phase
$\eta_{1,2}$: determined by extinction ratio
$v_{1,2}$: determined by modulator chirp and V_π
ψ: determined by bias, V_b

Laser phase and amplitude noise

2 The laser:

$$E_{\mathrm{laser}}(t) = E_0[1+\alpha_{\mathrm{RIN}}(t)]\exp\left[j\omega_c t + j\int_0^t \Delta\omega(t')dt'\right]$$

α_{RIN}: Laser amplitude noise (RIN)
$\Delta\omega$: Laser frequency noise
ω_c: optical carrier frequency

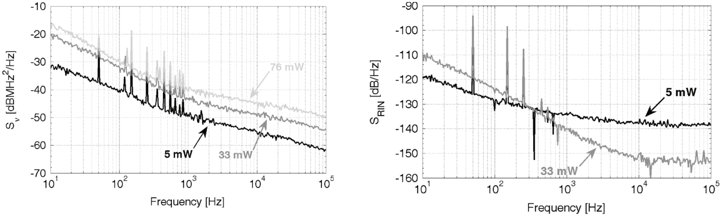


Figure: The laser frequency noise and RIN [2]

[2] K. Volyanskiy et al. J. Lightwave Technology. **28** 2730–5 (2010).

The optical path: the optical fiber

3 Optical propagation

The effects of dispersion and nonlinearity can be modeled by:

$$\frac{\partial E}{\partial z} = -\frac{\alpha}{2}E - \beta_1 \frac{\partial E}{\partial t} - i\frac{\beta_2}{2}\frac{\partial^2 E}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 E}{\partial t^3} + i\gamma|E|^2 E$$

α: fiber loss
γ: Kerr nonlinearity

$\beta_1 = 1/v_g$: group velocity
β_2: dispersion
β_3: 3rd order dispersion

The optical path: Detection

4 Detection:

The detected RF signal from the beating of optical harmonics:

$$V_{\mathrm{RF}}(t) = \rho R \sum_{m=-\infty}^{\infty} a_m(L,t) a^*_{m-1}(L,t) \exp(j\omega_0 t)$$

For a perfect fiber (only delay): [3]

$$V_{\mathrm{RF}}^{(\mathrm{ideal})}(t) = P_{\mathrm{opt}} R \rho \eta \cos\left(\frac{\pi V_B}{V_\pi}\right) J_1\left(\frac{\pi V_{\mathrm{in}}}{V_\pi}\right) \cos\left[\omega_0 t + \phi(t)\right]$$

ρ: photodetector responsivity

R: impedence P_{opt}: optical power

[3] X. S. Yao and L. Maleki, JOSA B, **8** 1725–35 (1996).

Optical propagation: effect on the signal

The optical signal is affected by loss, dispersion and nonlinearity.

1. Increasing nonlinearity increases the power transferred to harmonics further from the carrier.
2. The phase of the harmonics rotates leading to reduction of the detected signal

Ignoring the noise, these changes in the harmonics are given by:

$$\frac{\partial a_m}{\partial z} = -\frac{1}{2}\alpha a_m - j\frac{\beta_2}{2}(m\omega_0)^2 a_m + j\gamma \sum_{k=-M}^{M} \sum_{l=-M}^{M} a_j a_k a_{j+k-m}^*$$

Nonlinearity: Power transfer to higher harmonics

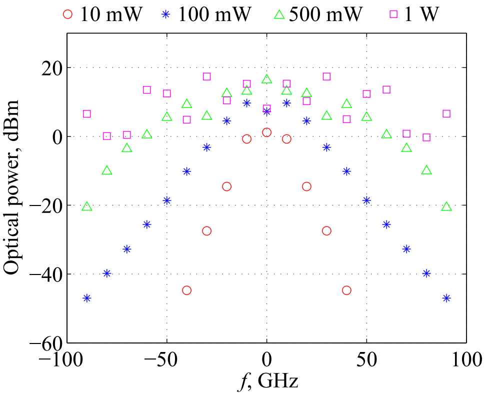


Figure: Theoretical optical power in the harmonics for a 10 GHz OEO after 6 km of transmission through SMF-28.

Dispersion and dephasing

The phase of harmonics is changed by dispersion and nonlinearity:

$$a_m(z) = a_m(0)\exp\left[-\frac{1}{2}\alpha z + j\theta_m(z)\right]$$

Phase differences between harmonics reduce the detected signal:

$$\delta = \frac{\theta_1(L) + \theta_{-1}(L)}{2} - \theta_0(L)$$

$$V_{\mathrm{RF}}(t) = \exp(-\alpha L)\cos(\delta)\, V_{\mathrm{RF}}^{(\mathrm{ideal})}(t)$$

For dispersion with an ideal modulator: $\delta = \dfrac{\beta_2}{2}\omega_0 L$

Optical transmission: the signal

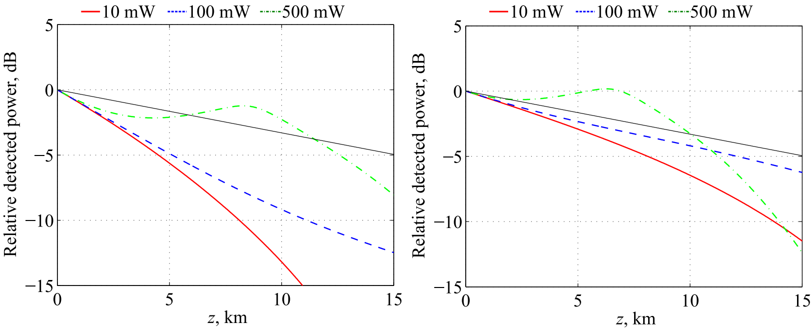


Figure: Calculated detected power for a 10 GHz OEO for a modulator with (a) zero chirp, and (b) chirp of $\alpha = 0.6$

Optical propagation: Dispersion

First, looking at dispersion alone we have

$$\frac{\partial A}{\partial z} + \beta_2 \Delta\omega(t)\partial_t A = -j\frac{\beta_2}{2}\left[\partial_t^2 - \Delta\omega(t)^2\right]A,$$

1. Dispersion converts laser frequency noise to timing jitter
2. This is equivalent to a phase noise of the RF signal of

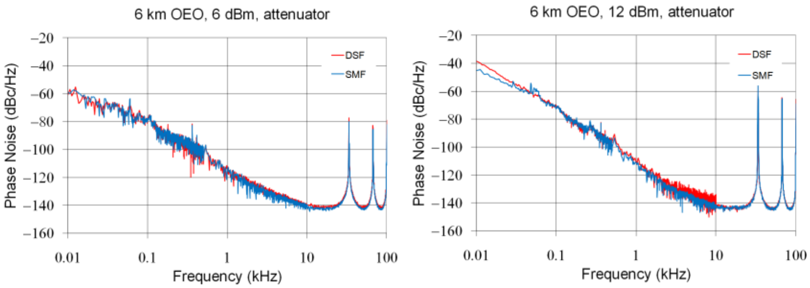
$$\phi_{\mathrm{RF}}(z) = \beta_2 \omega_0 \Delta\omega(t) z,$$

3. This has recently been shown by Volyanskiy et al. [4]
4. The right hand side terms only effect the phase of the harmonics

[4] K. Volyanskiy et al. J. Lightwave Technology. **28** 2730–5 (2010).

Experimental evidence: not just dispersion!

- Using low dispersion fiber (DSF) has no effect on measured RF flicker noise.
- A significant power dependence is seen



- Does the Kerr effect contribute effect the RF phase noise?

Optical propagation: Nonlinearity

$$\frac{\partial A}{\partial z} = j\gamma(1 + 2\alpha_{\mathrm{RIN}})|A|^2 A$$

In the presence of nonlinearity alone, the signal only experiences nonlinear phase rotation. This has no effect after direct detection.

$$A(z,t) \approx \exp\left[j\gamma(1 + 2\alpha_{\mathrm{RIN}})|A(z,0)|^2\right] A(0,t)$$

$$V_{\mathrm{RF}}(t) \propto |A(z,t)|^2 = |A(0,t)|^2$$

However, the combination of nonlinearity and dispersion can have complex effects.

Noise exchange between harmonics

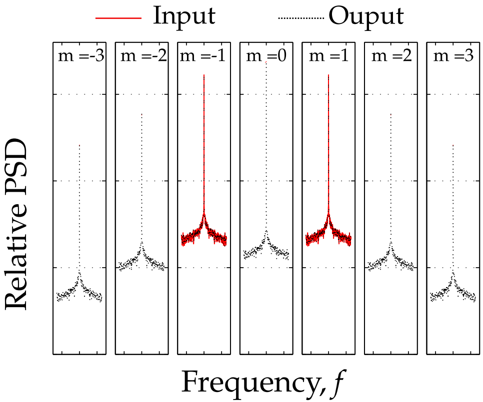


Figure: The theoretical RF frequency and amplitude noise converted from a typical LFN spectrum by dispersion for a SMF 28 fiber.

Parametric amplification

- Amplitude noise is parametrically amplified
- RF phase noise is not affected by nonlinearity

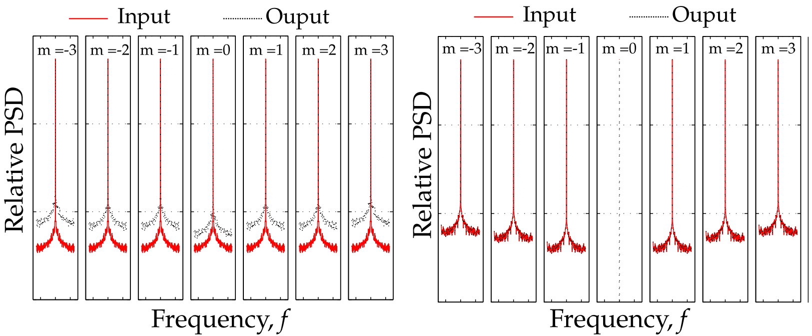


Figure: The theoretical optical spectra with initial (a) laser amplitude and (b) RF phase noise modulated onto the carrier.

Optical propagation and noise

We explicitly put the laser frequency noise into the field

$$E(z,t) = A(z,t)\left[1 + \alpha_{\mathrm{RIN}}\right] \exp\left[j\int_0^t \Delta\omega(t')dt'\right]$$

This gives the equation for the evolution of the RF harmonics, including the effects of laser frequency noise:

$$\frac{\partial A(z,t)}{\partial z} \simeq -\frac{1}{2}\alpha A - j\frac{\beta_2}{2}\left[\tfrac{\partial}{\partial t} + j\Delta\omega\right]^2 A - \frac{\beta_3}{6}\left[\tfrac{\partial}{\partial t} + j\Delta\omega\right]^3 A + j\gamma\left[1 + 2\alpha_{\mathrm{RIN}}\right]|A|^2 A$$

Effect on noise: Laser phase noise

- Dispersion converts laser frequency noise to RF phase noise

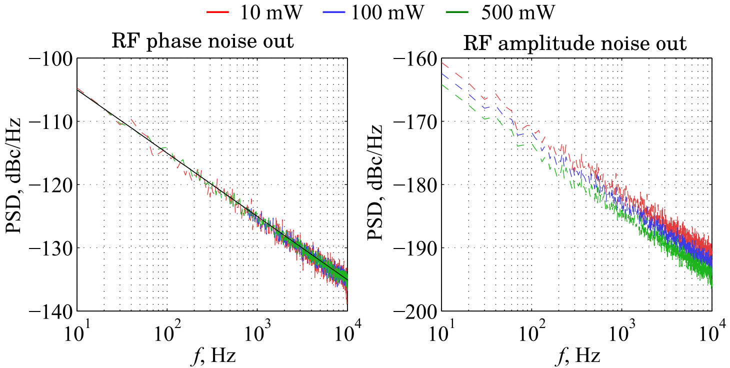


Figure: The theoretical detected RF frequency and amplitude noise converted from a typical laser frequency noise spectrum by dispersion and nonlinearity.

Effect on noise: Laser amplitude noise

- RIN is parametrically amplified but only at high powers
- Kerr nonlinearity and third order dispersion converts RIN to negligible RF phase noise

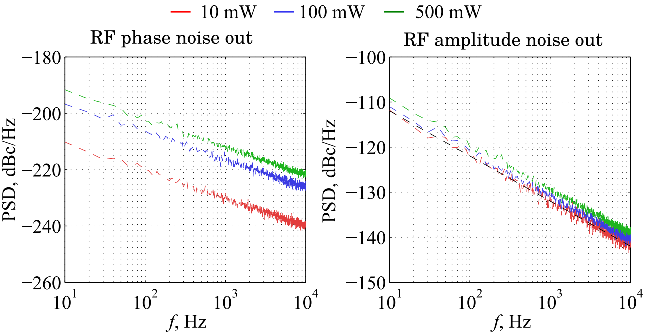


Figure: The theoretical detected RF phase and amplitude noise spectra after optical propagation with a typical RIN input.

Effect on noise: RF phase noise

- Kerr nonlinearity does not affect RF phase noise

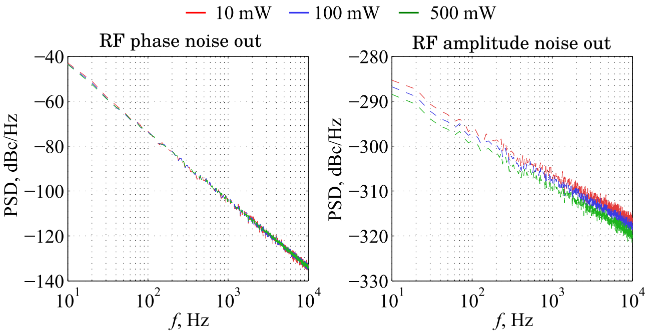


Figure: The theoretical detected RF phase and amplitude noise spectra after optical propagation with an RF phase noise input

Effect on noise: RF amplitude noise

- Kerr nonlinearity and third order dispersion converts RF amplitude noise to negligible RF phase noise

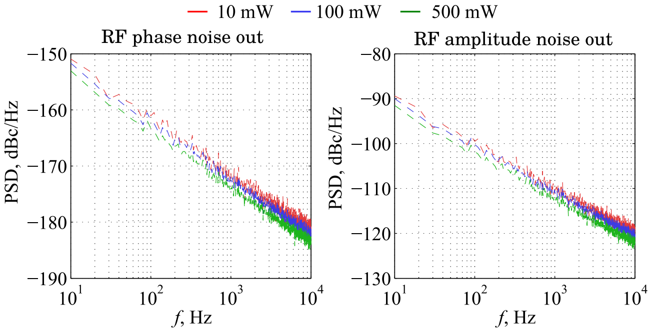


Figure: The theoretical detected RF phase and amplitude noise spectra after optical propagation with an RF amplitude noise input

Conclusions

1. We are conducting a systematic investigation of the optical domain portion of OEOs
2. We have investigated the effects of dispersion and nonlinearity on signal and noise
3. Kerr nonlinearity was not found to be a cause of length-dependent RF phase noise
4. We are investigating other nonlinear amplification processes in the fiber, in particular Brillouin and Rayleigh effects